Technische Universität München
Institut für Informatik
Scientific Computing in Computer Science

WS 2011/12
Worksheet
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Fundamental Algorithms 1

Exercise 1

Prove (by induction over n) that $\frac{1}{3}n^2 + 5n + 30 \in O(n^2)$ for all $n \in \mathbb{N}^+$.

Exercise 2

(a) Compare the growth of the following functions using the o-, O-, and Θ-notation:

1. $n \log n$
2. n^l for all $l \in \mathbb{N}$
3. 2^n

Hint: use de l'Hôpital's rule for asymptotically larger or smaller!

(b) Try to give a simple characterization of the growth of the following expressions using the Θ-notation:

$$1) \sum_{i=1}^{n} \frac{1}{i} \qquad\qquad 2) \log(n!)$$

Hint for $\log(n!)$: try to prove $n^{\frac{n}{2}} \leq n! \leq n^n$ first!

Exercise 3

Let $l(x)$ be the number of bits of the representation of x in the binary system. Prove:

$$\sum_{i=1}^{n} l(i) \in \Theta(n \log n)$$

Exercise 4

Prove that Θ defines an equivalence relation on the set of functions $\{f \mid f : \mathbb{N} \to \mathbb{R}\}$. Use that $(f, g) \in \Theta \Leftrightarrow f \in \Theta(g)$