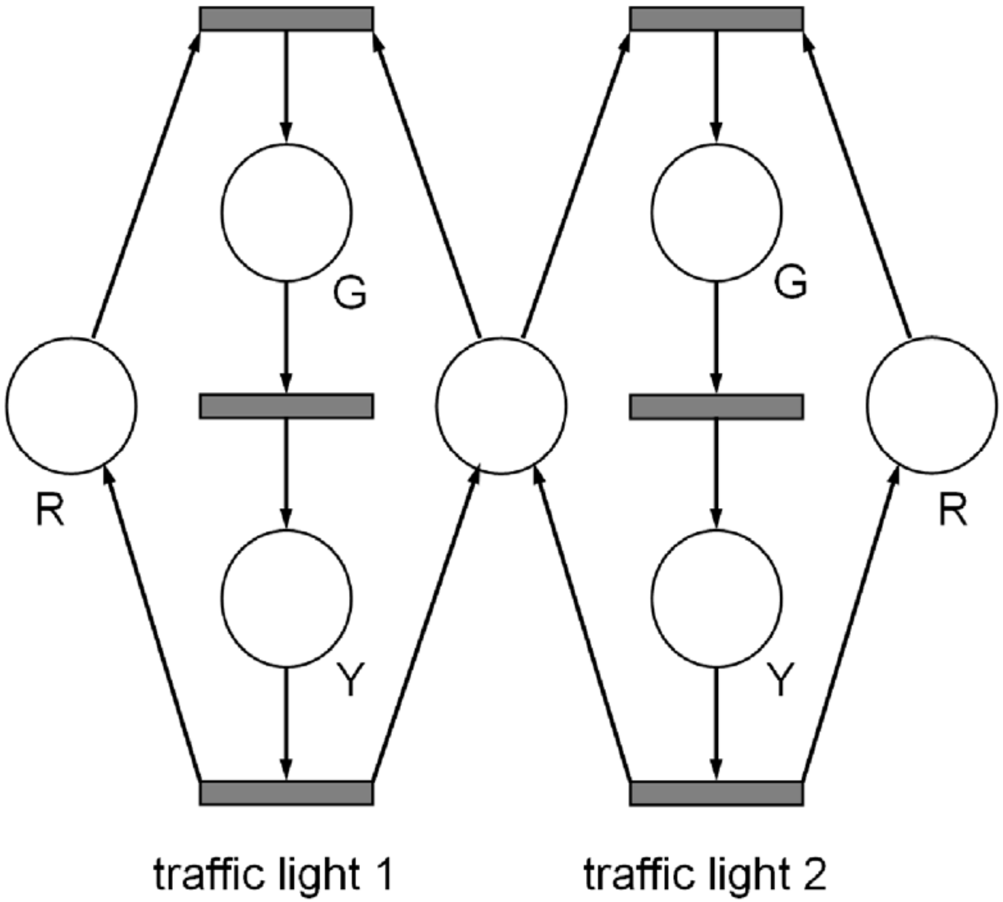
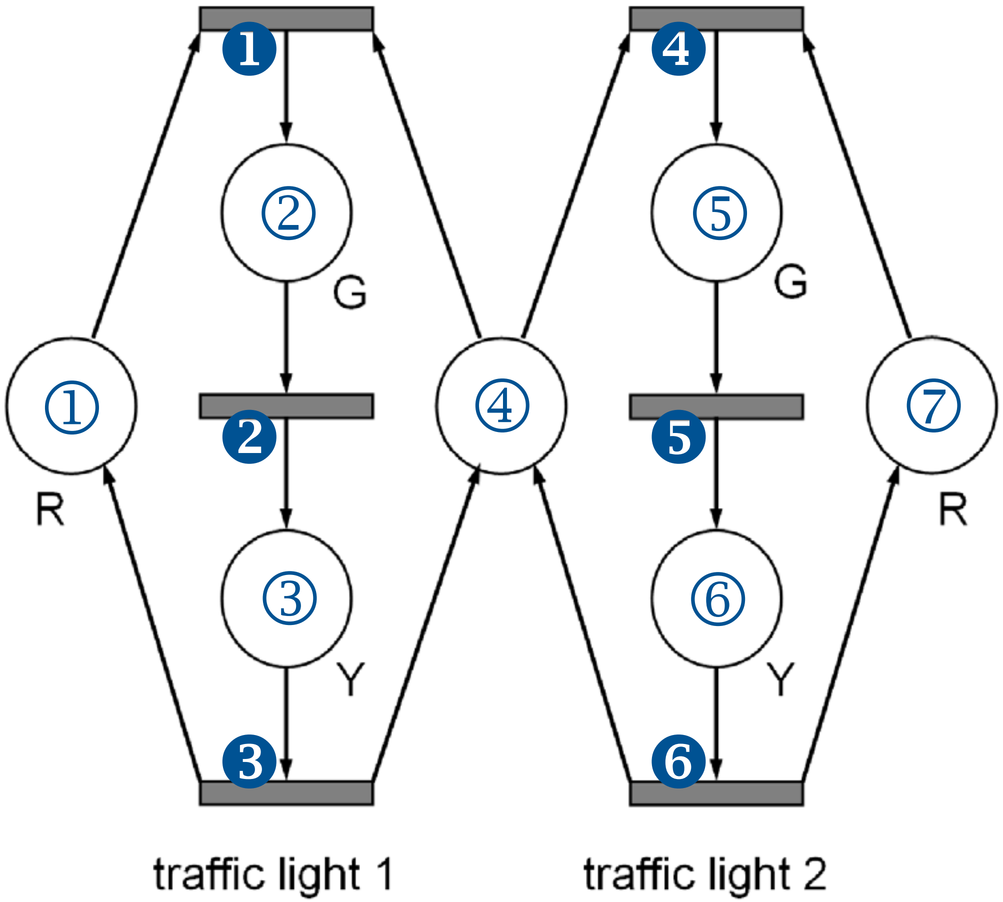
- Exercise 1: PETRI Networks
  - Given



- Exercise 1: PETRI Networks
  - Given



$I =$ (matrix: places $s_1 - s_7$ × transitions $t_1 - t_6$)

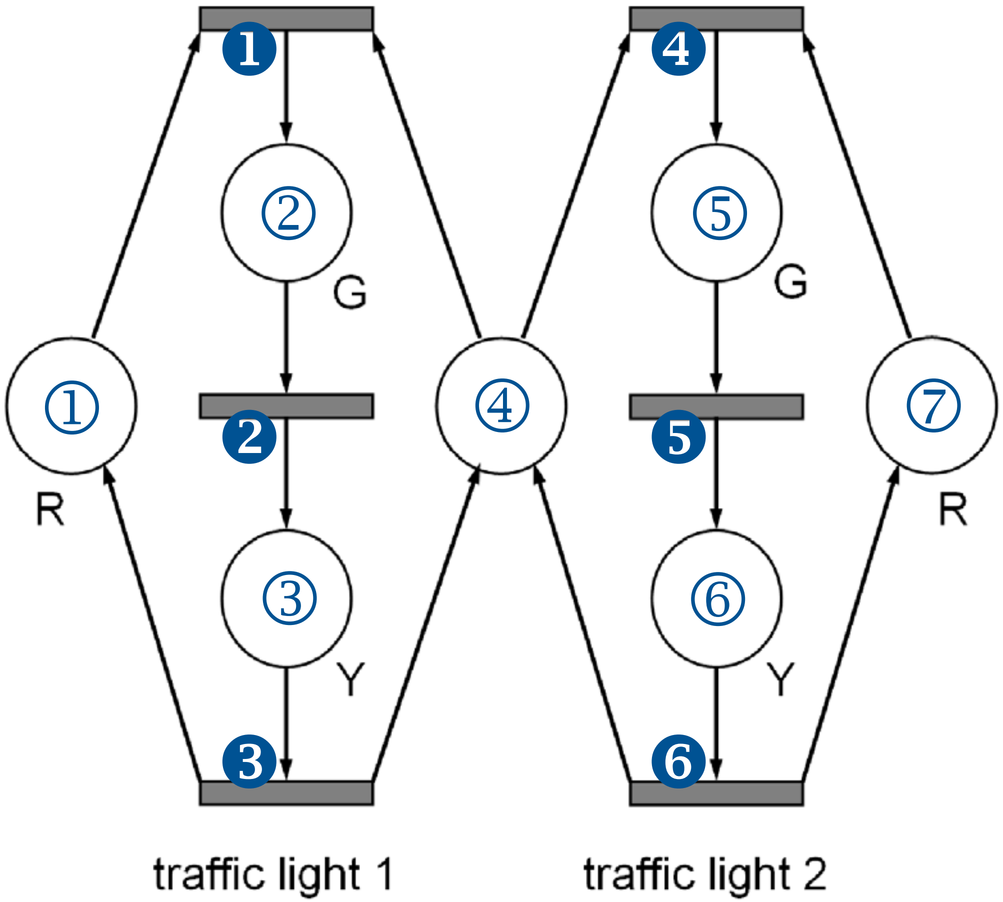
places $s_1 - s_7$

transitions $t_1 - t_6$

$\kappa = (①,②,③,④,⑤,⑥,⑦)^T$

$\Psi = (❶,❷,❸,❹,❺,❻)^T$

- Exercise 1: PETRI Networks
  - Given

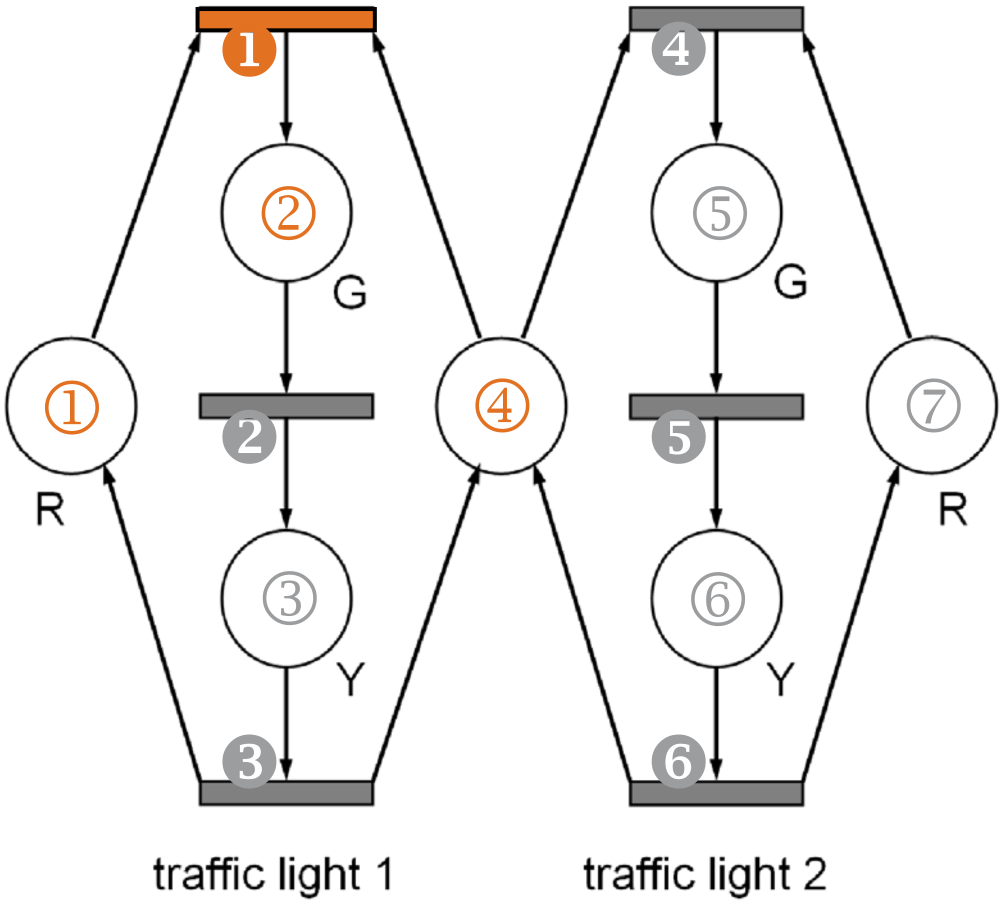


$$I = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix}$$

$\kappa = (①,②,③,④,⑤,⑥,⑦)^T$

$\Psi = (❶,❷,❸,❹,❺,❻)^T$

- Exercise 1: PETRI Networks
  - Given

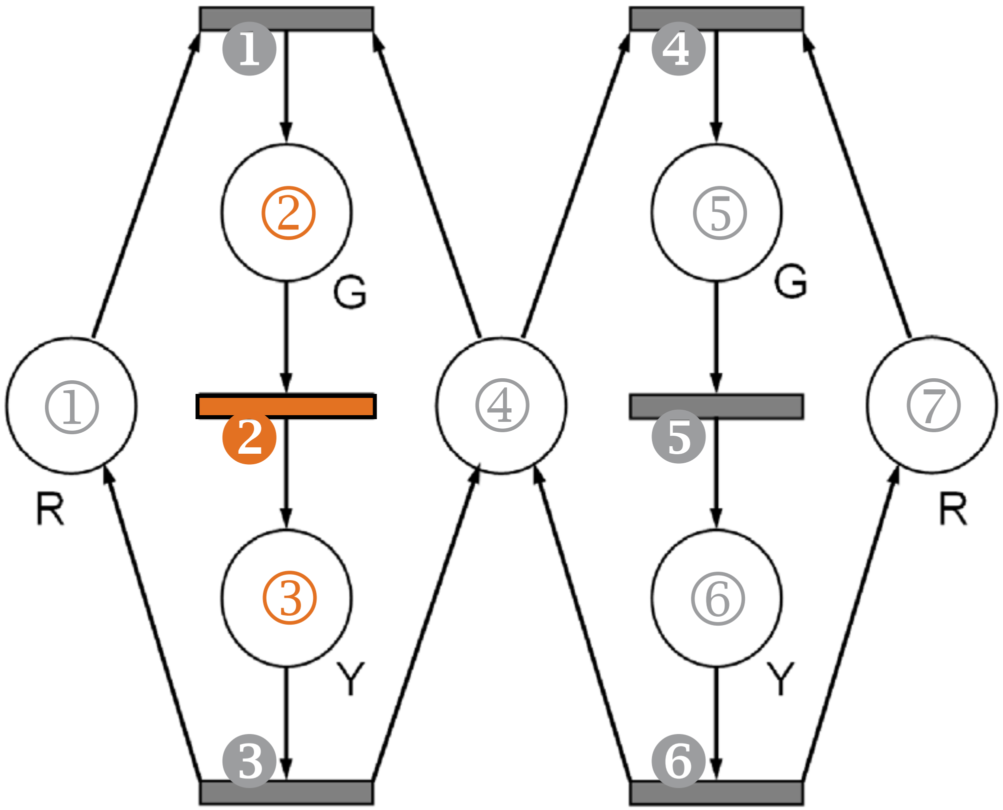


$$I = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix}$$

$$\kappa = (①,②,③,④,⑤,⑥,⑦)^T$$

$$\Psi = (❶,❷,❸,❹,❺,❻)^T$$

- Exercise 1: PETRI Networks
  - Given



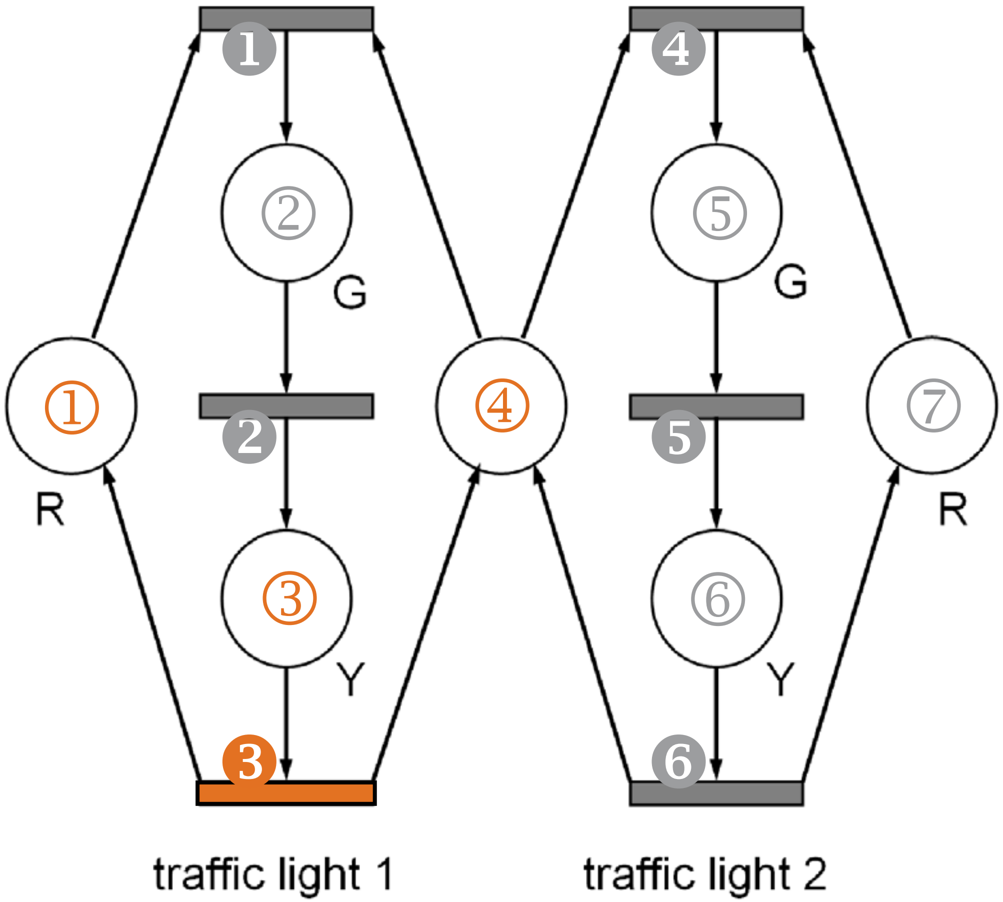
traffic light 1 traffic light 2

$$I = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix}$$

$$\kappa = (①,②,③,④,⑤,⑥,⑦)^T$$

$$\Psi = (❶,❷,❸,❹,❺,❻)^T$$

- Exercise 1: PETRI Networks
  - Given

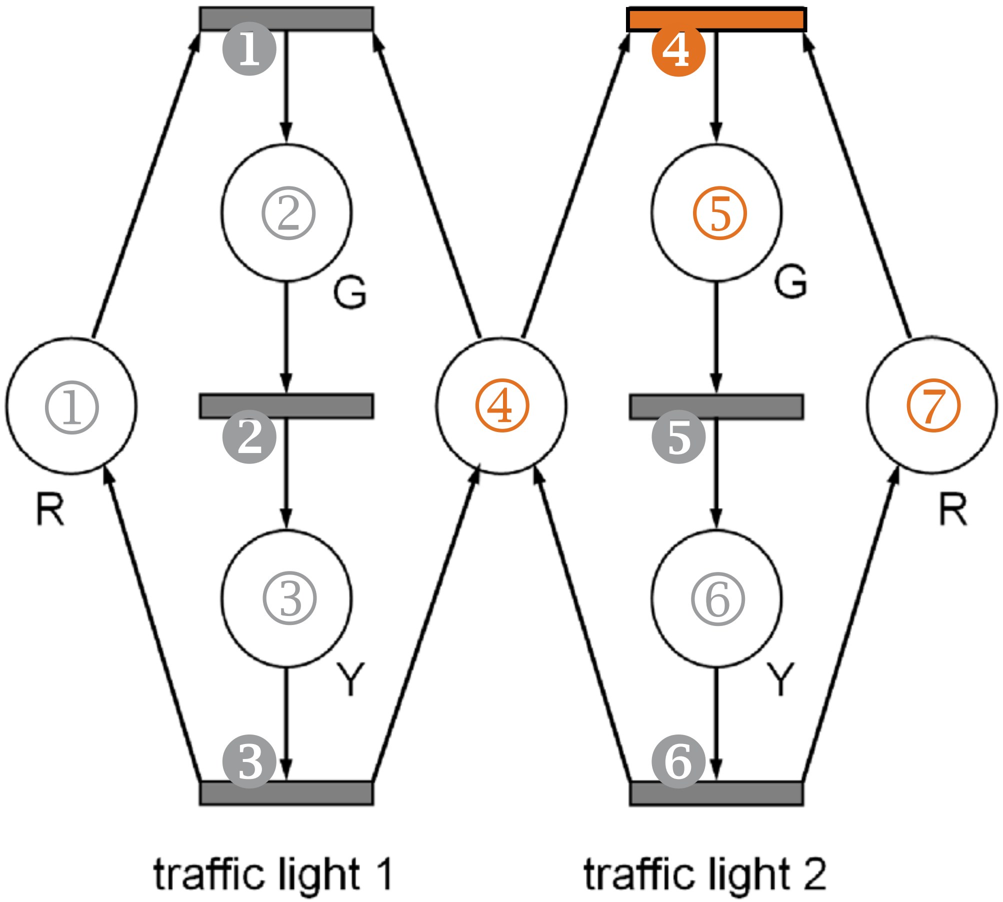


$$I = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix}$$

$\kappa = (①,②,③,④,⑤,⑥,⑦)^T$

$\Psi = (❶,❷,❸,❹,❺,❻)^T$

- Exercise 1: PETRI Networks
  - Given

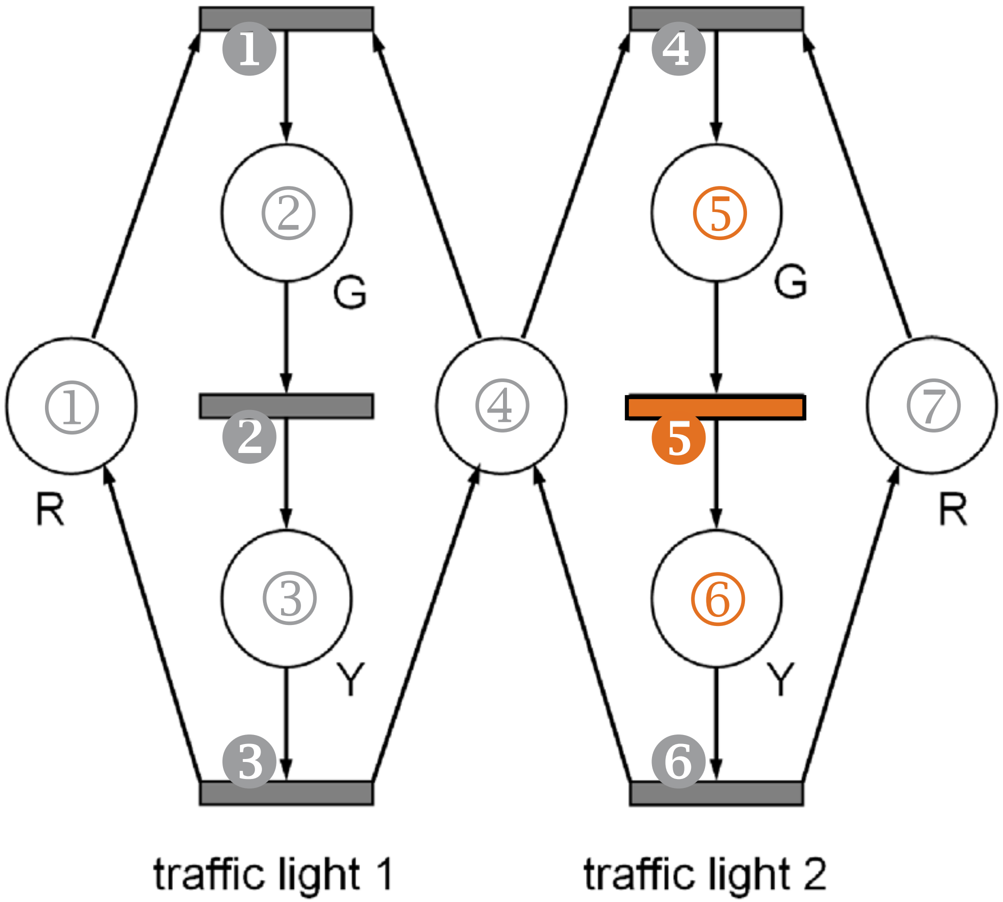


$$I = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix}$$

$$\kappa = (①,②,③,④,⑤,⑥,⑦)^T$$

$$\Psi = (❶,❷,❸,❹,❺,❻)^T$$

- Exercise 1: PETRI Networks
  - Given

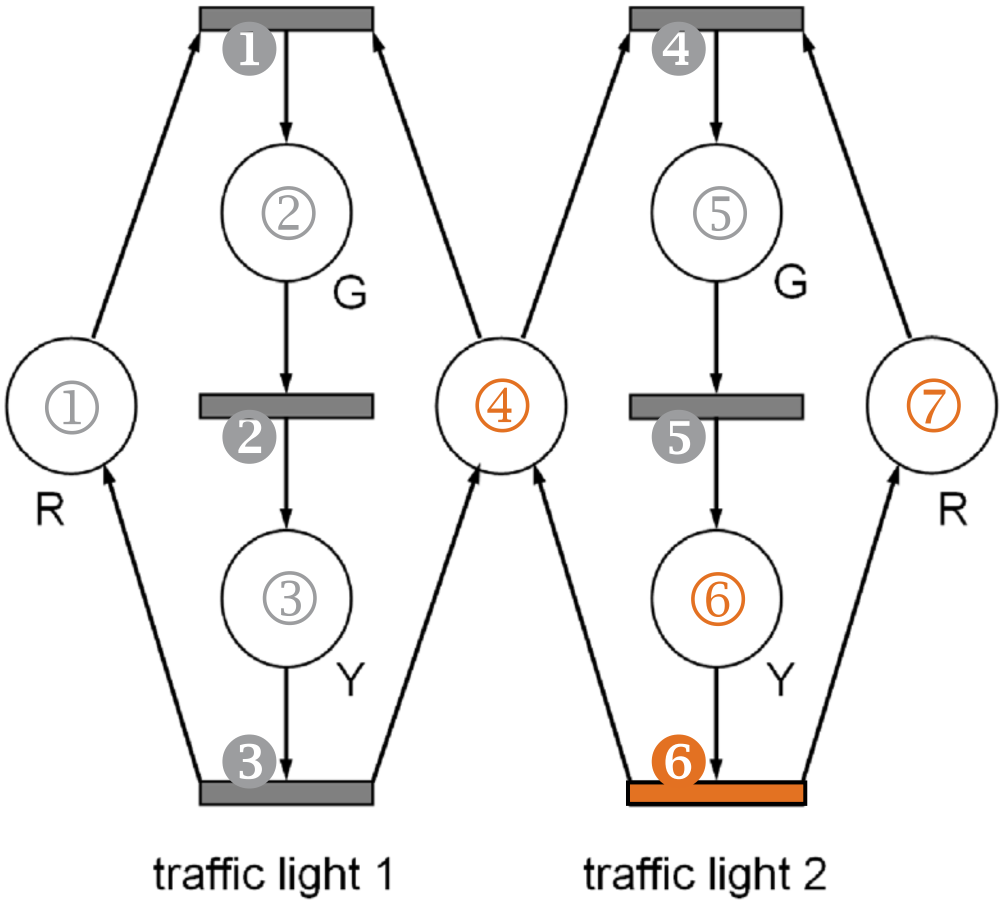


$$I = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix}$$

$\kappa = (①,②,③,④,⑤,⑥,⑦)^T$

$\Psi = (❶,❷,❸,❹,❺,❻)^T$

- Exercise 1: PETRI Networks
  - Given

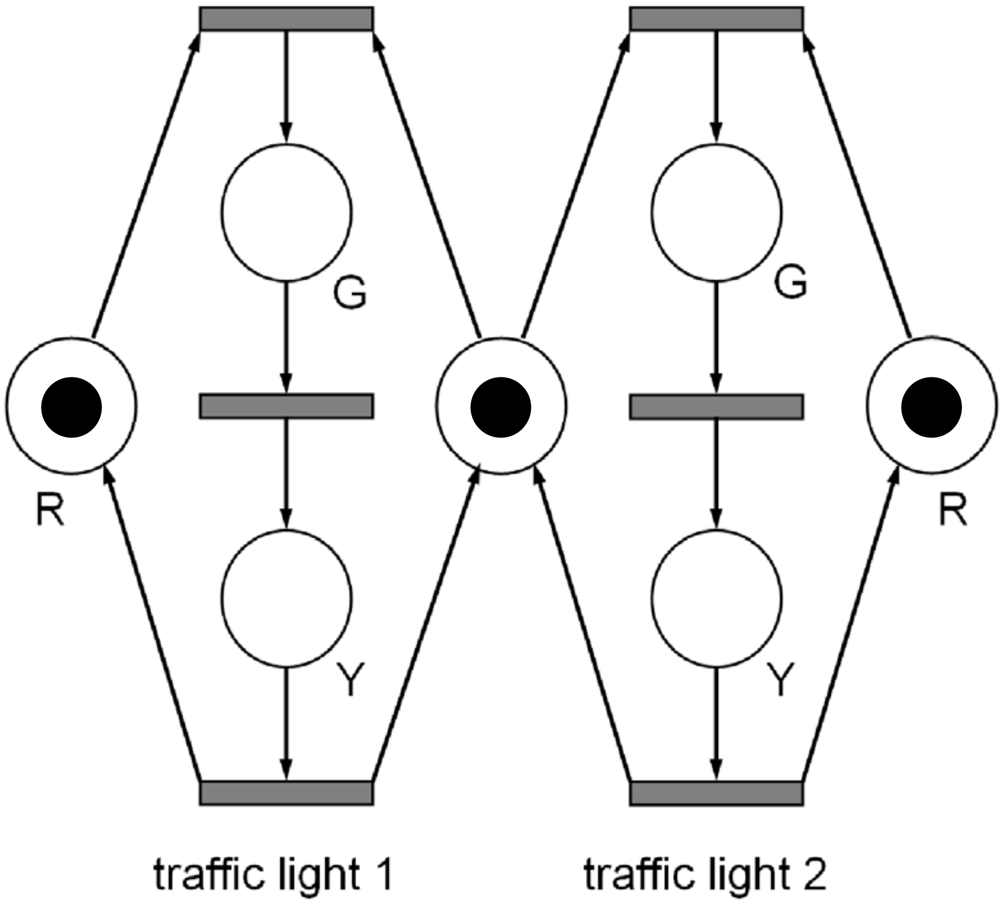


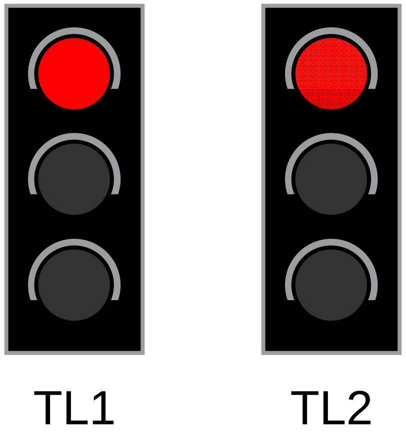
$$I = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix}$$

$$\kappa = (①,②,③,④,⑤,⑥,⑦)^T$$

$$\Psi = (❶,❷,❸,❹,❺,❻)^T$$

- Exercise 1: PETRI Networks
  - Part a) sketch one full switching cycle

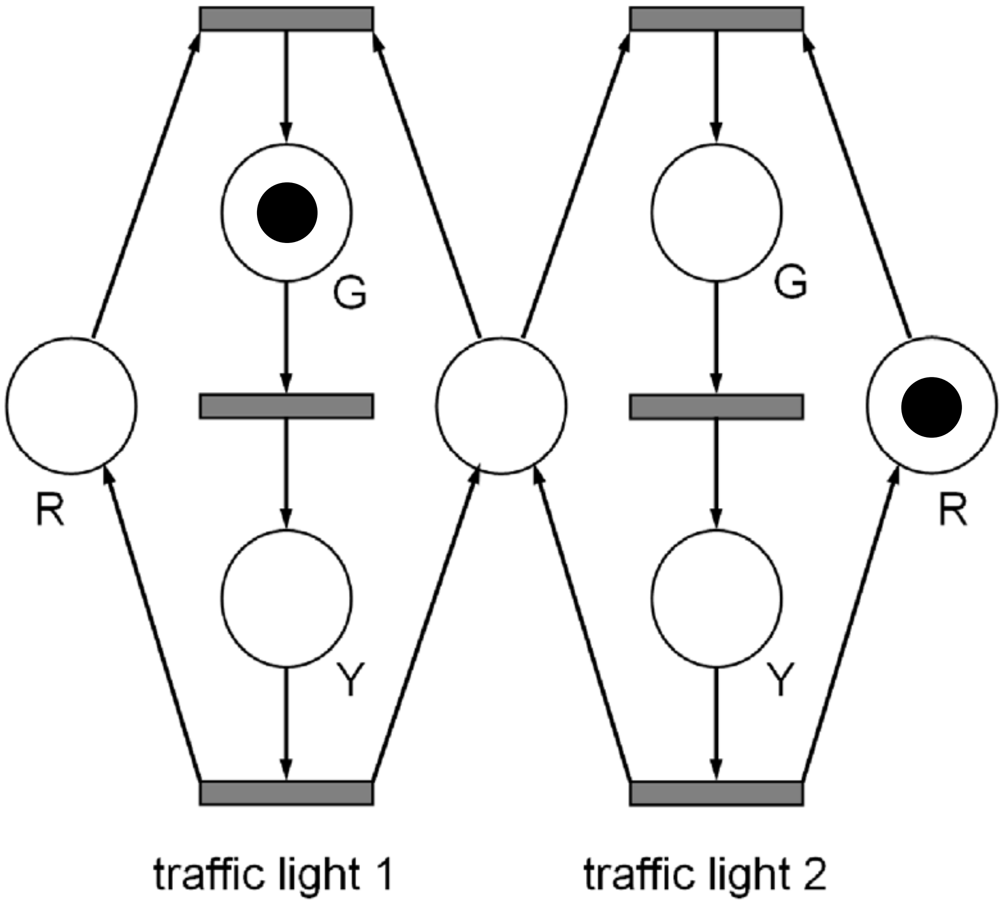


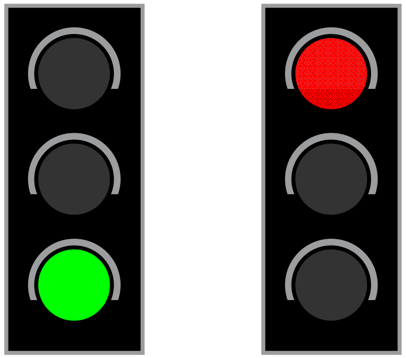


$\kappa = (1, 0, 0, 1, 0, 0, 1)^T$

$\Psi = (1, 0, 0, 0, 0, 0)^T$

- Exercise 1: PETRI Networks
  - Part a) sketch one full switching cycle

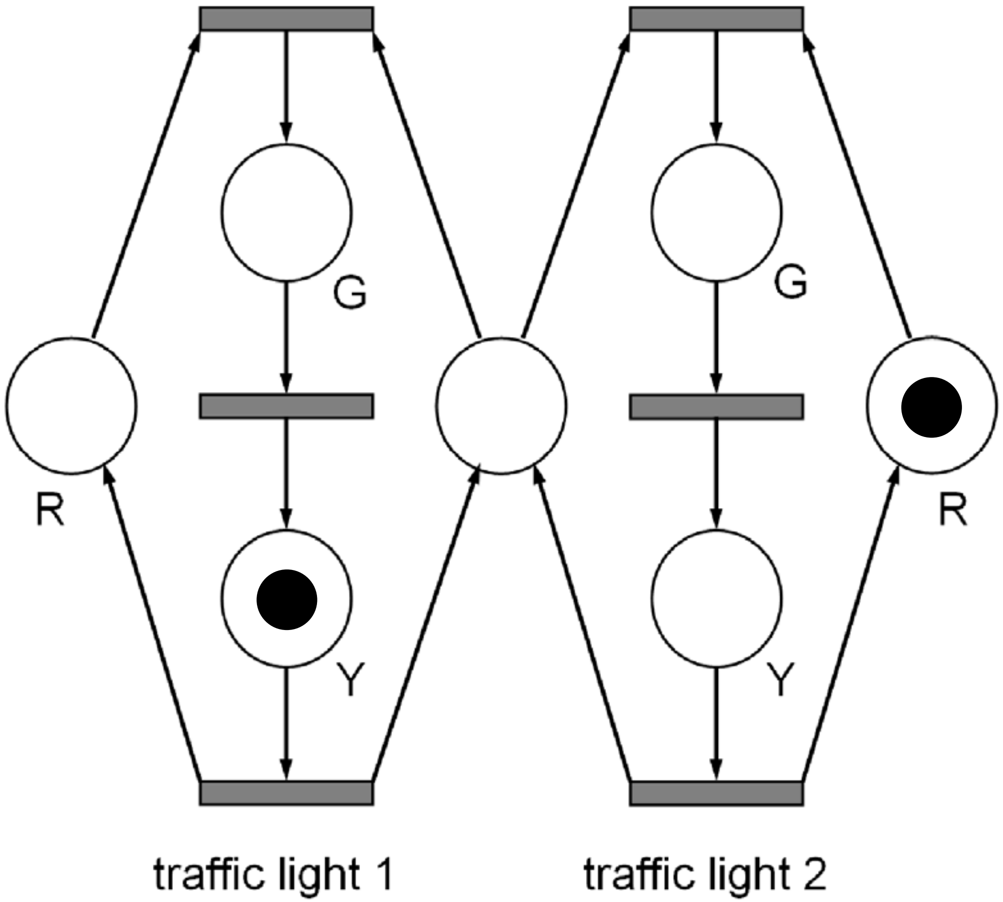


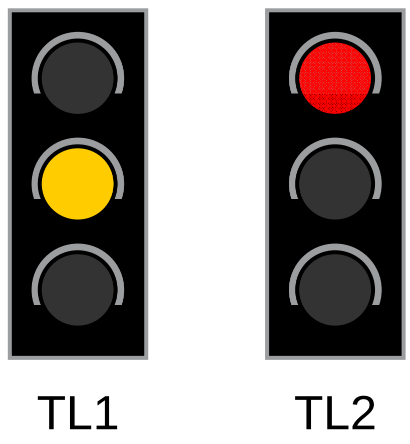


$\kappa = (0, 1, 0, 0, 0, 0, 1)^T$

$\Psi = (0, 1, 0, 0, 0, 0)^T$

- Exercise 1: Petri Networks
  - Part a) sketch one full switching cycle



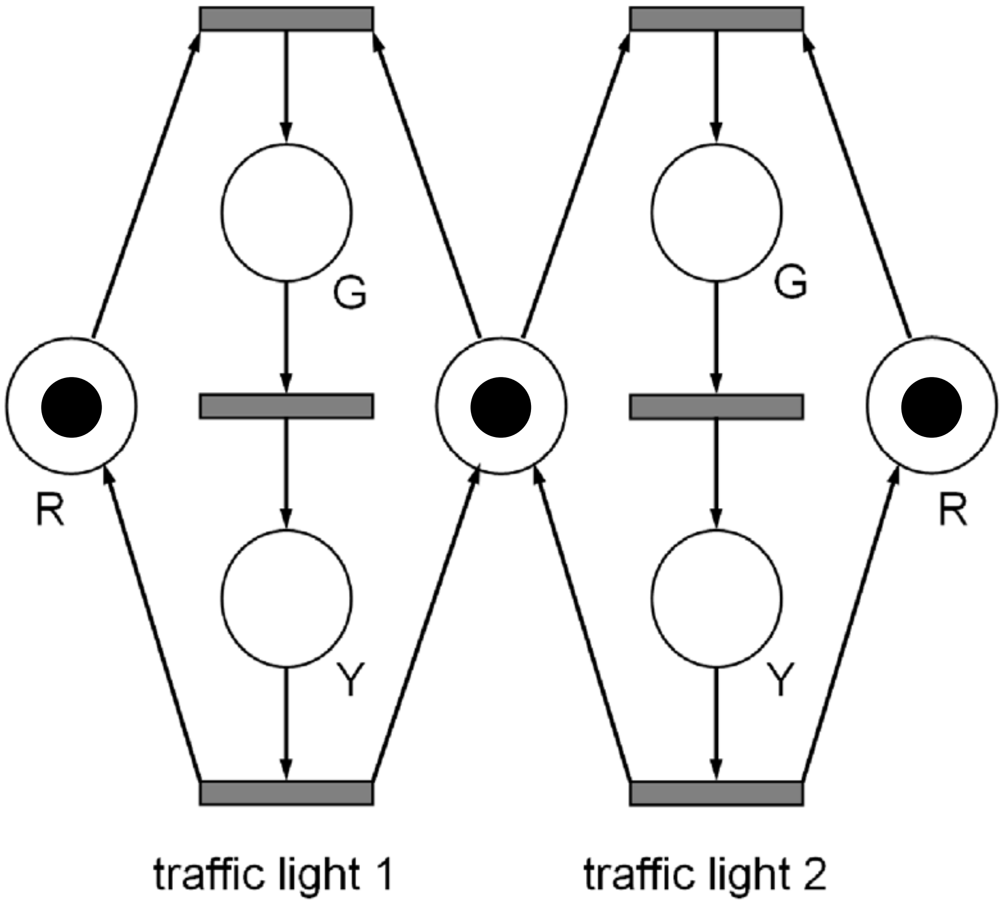


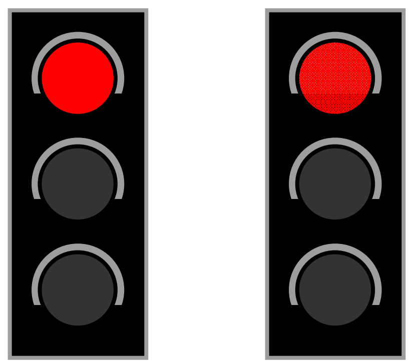
$\kappa = (0, 0, 1, 0, 0, 0, 1)^T$

$\Psi = (0, 0, 1, 0, 0, 0)^T$

Exercise 1: PETRI Networks

- Part a) sketch one full switching cycle

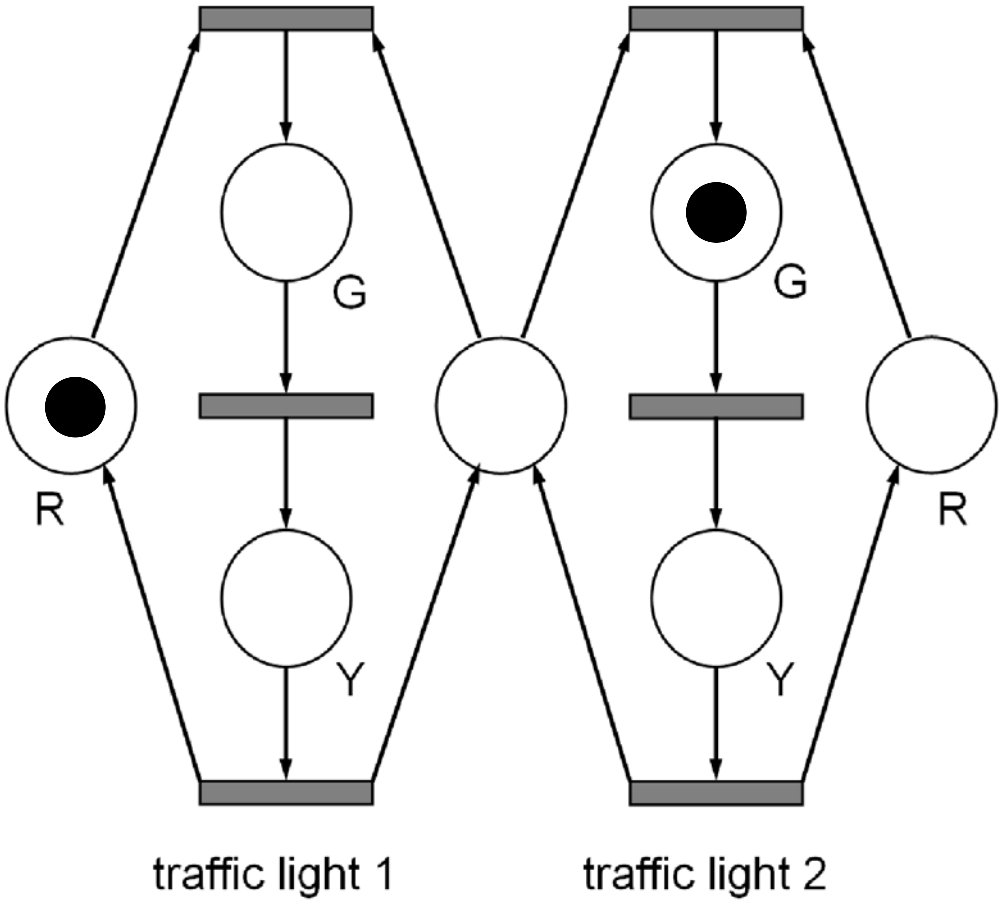


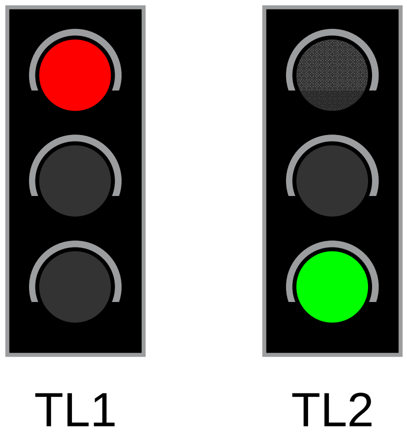


$\kappa = (1, 0, 0, 1, 0, 0, 1)^T$

$\Psi = (0, 0, 0, 1, 0, 0)^T$

- Exercise 1: PETRI Networks
  - Part a) sketch one full switching cycle

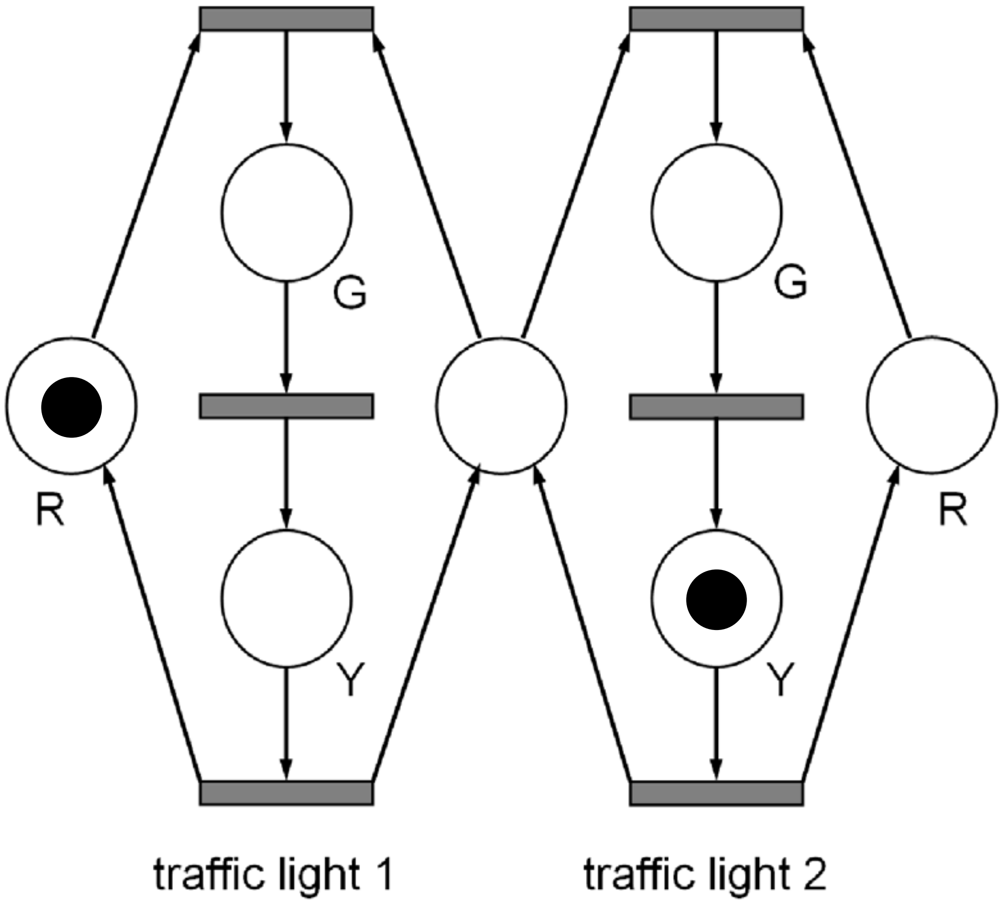


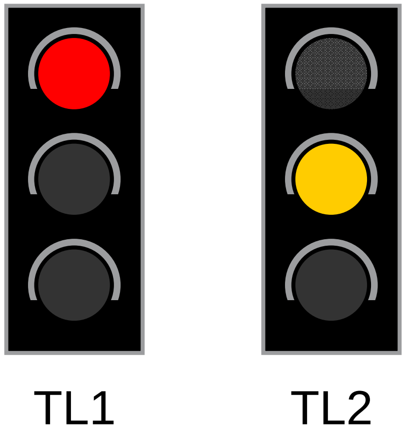


$\kappa = (1, 0, 0, 0, 1, 0, 0)^T$

$\Psi = (0, 0, 0, 0, 1, 0)^T$

- Exercise 1: PETRI Networks
  - Part a) sketch one full switching cycle

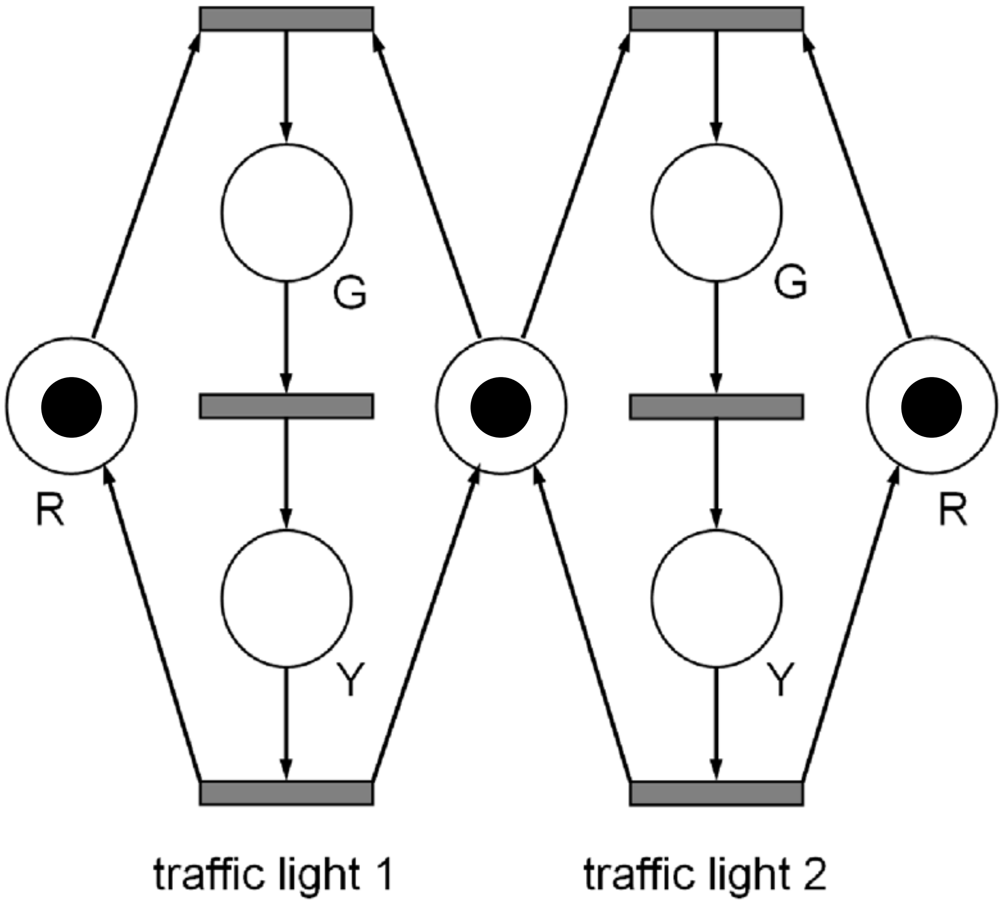


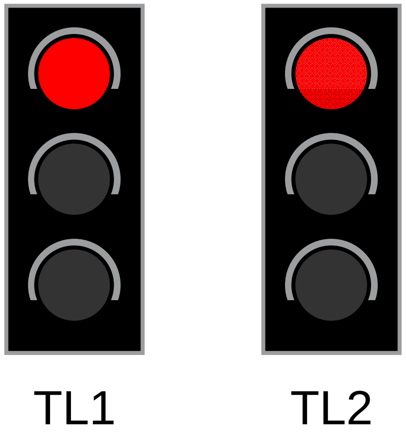


$\kappa = (1, 0, 0, 0, 0, 1, 0)^T$

$\Psi = (0, 0, 0, 0, 0, 1)^T$

- Exercise 1: PETRI Networks
  - Part a) sketch one full switching cycle





$\kappa = (1, 0, 0, 1, 0, 0, 1)^T$

$\Psi = (0, 0, 0, 0, 0, 0)^T$

- Exercise 1: PETRI Networks
  - Part a) are there any configurations that are not reachable
  - ➔ any configuration κ' is reachable if the equation

$$I \cdot \Psi = \kappa' - \kappa \qquad (1)$$

  has a solution in IN, with κ denoting a previous configuration

  - possible unreachable configurations
    - $\kappa = (0, 1, 0, 0, 1, 0, 0)^T$, i.e. Green–Green
    - $\kappa = (0, 1, 0, 0, 0, 1, 0)^T$, i.e. Green–Yellow
    - $\kappa = (0, 0, 1, 0, 1, 0, 0)^T$, i.e. Yellow–Green
    - $\kappa = (0, 0, 1, 0, 0, 1, 0)^T$, i.e. Yellow–Yellow
  - ➔ check for the above configurations if starting from κ_0 the solution of (1) leads to an illegal Parikh vector Ψ with elements $\psi_i \notin$ IN

- Exercise 1: PETRI Networks
  - Part a) are there any configurations that are not reachable
  - example: $\kappa' = (0, 0, 1, 0, 0, 0, 1)^T$, i.e. Yellow–Red

$$I \cdot \Psi = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \\ \psi_6 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \kappa' - \kappa_0$$

$$\Psi = (1, 1, 0, 0, 0, 0)^T$$

➔ $\Psi = \Psi_1 + \Psi_2$ is a combination of two vectors $\Psi_1 = (1, 0, 0, 0, 0, 0)^T$ and $\Psi_2 = (0, 1, 0, 0, 0, 0)^T$, i.e. transitions R–R → G–R → Y–R

- Exercise 1: PETRI Networks
  - Part a) are there any configurations that are not reachable

  - Green–Green: $\kappa' = (0, 1, 0, 0, 1, 0, 0)^T$

$$I \cdot \Psi = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \\ \psi_6 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \\ -1 \\ 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \kappa' - \kappa_0$$

➔ factorisation of $(\Psi \mid \kappa' - \kappa_0)$ via GAUSSIAN elimination

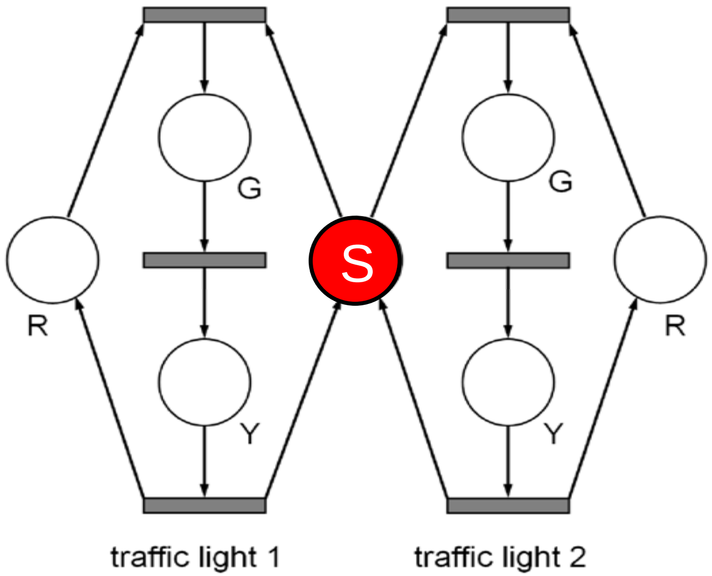
- Exercise 1: PETRI Networks
  - Part a) are there any configurations that are not reachable
  - Green–Green: $\kappa' = (0, 1, 0, 0, 1, 0, 0)^T$

$$\left(\begin{array}{cccccc|c} -1 & 0 & 1 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & -1 \end{array}\right) \rightarrow \left(\begin{array}{cccccc|c} -1 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

➔ for configuration Green–Green exists no solution, thus, the configuration $\kappa' = (0, 1, 0, 0, 1, 0, 0)^T$ is unreachable

  - the same is true for configurations G–Y, Y–G, and Y–Y

- Exercise 1: PETRI Networks
  - Part b) pseudo code implementation



if neglecting the switching from R → G → Y then just one semaphore *S* necessary

traffic light 1:

```
while (TRUE) do
  P(S)
    switch R,G,Y
  V(S)
od
```

traffic light 2:

```
while (TRUE) do
  P(S)
    switch R,G,Y
  V(S)
od
```

Questions?