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High Performance Computing - Programming Paradigms and Scalability

Exercise Sheet 2: Networks

1 Network Evaluation: Diameter, Bisection Width, and Cost

Network evaluation is based on different parameters. For instance, the diameter of a network is the *maximum distance* between any two processing nodes in the network, where the distance between two processing nodes is defined as the shortest path (in terms of number of edges) between them. The bisection width of a network is defined as the *minimum amount of edges* that have to be removed in order to separate the network into two equal parts.

a) Calculate the diameter and the bisection width, depending on the maximum amount of nodes N and the dimension d only, for the following network topologies:

   i) linear array (chain)
   ii) ring
   iii) binary tree
   iv) d-dimensional torus
   v) d-dimensional hypercube

b) Calculate the costs – defined as number of edges – for the network topologies described in a), depending on the maximum amount of nodes N and the dimension d only.

c) Compare the above topologies according to diameter, bisection width, and costs and explain which one has the best cost-benefit ratio.

2 Performance Evaluation: Torus vs. Hypercube

The bisection bandwidth of a network is the maximum transmission performance over the bisection line, i.e. the sum of all single channel bandwidths from all edges that are "cut" via the bisection line.

a) Given is a 3-dimensional torus with size 8×8×16 (i.e. 1024 nodes). Furthermore, every edge (i.e. cable) of this network has a bandwidth of 400 MBps. Every message sent through this network – assuming there is no congestion – has a latency of 0.5 ns to be passed from one node to another.

   i) Calculate the bisection bandwidth of this network.

   ii) Calculate the diameter of this network.

   iii) Calculate the maximum delay of this network.

b) Given is a 10-dimensional hypercube (i.e. 1024 nodes). Furthermore, every edge (i.e. cable) of this network has a bandwidth of 120 MBps. Every message sent through this network – assuming there is no congestion – has a latency of 0.7 ns to be passed from one node to another.

   i) Calculate the bisection bandwidth of this network.

   ii) Calculate the diameter of this network.

   iii) Calculate the maximum delay of this network.

c) Calculate the cost of both networks described in a) and b) in order to make a decision dependent on a cost-benefit ratio which topology to choose for a 1024 processor machine concerning bisection bandwidth, delay, and cost. Discuss your decision!

3 Shortest Path Routing

a) Given is a 2-dimensional mesh with size $M \times N$, the nodes are labeled from 1 to M in x-direction and from 1 to N in y-direction. Consider two arbitrary nodes n_1 and n_2 with coordinates (x_1, y_1) and (x_2, y_2), resp. How many different shortest paths from node n_1 to node n_2 do exist? Give a general formula for calculating the amount of shortest paths for two arbitrary nodes n_1 and n_2, depending on x_1, y_1, x_2, y_2, M, and N only.

b) Given is a d-dimensional hypercube. Consider two arbitrary nodes n_1 and n_2 with coordinates $v_1^1, v_2^1, \ldots v_{d-1}^1, v_d^1$ and $v_1^2, v_2^2, \ldots v_{d-1}^2, v_d^2$, resp. How many different shortest paths from node n_1 to node n_2 do exist? Give a general formula for calculating the amount of shortest paths for two arbitrary nodes n_1 and n_2, depending on d, $v_1^1, v_2^1, \ldots v_{d-1}^1, v_d^1$ and $v_1^2, v_2^2, \ldots v_{d-1}^2, v_d^2$ only.