High Performance Computing – Programming Paradigms and Scalability

Part 3: Foundations

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Summer Term 2015

Overview

- terms and definitions
- process interaction for MemMS
- process interaction for MesMS
- example of a parallel program
- load balancing
- state-of-the-art LB strategies

A distributed system is the one
that prevents you from working because of the failure
of a machine that you had never heard of.
—Leslie Lamport

Terms and Definitions

- general design questions
  - several considerations have to be taken into account for writing a parallel program (either from scratch or based on an existing sequential program)
  - standard questions comprise
    - which part of the (sequential) program can be done in parallel
    - where / how often does synchronisation become necessary
    - which programming model to be used
    - what kind of (data) structure to be used for parallelisation
    - what about load balancing strategies
    - what kind of architecture is the target machine
    - …

Terms and Definitions

- dependence analysis
  - processes / (blocks of) instructions cannot be executed simultaneously if there exist dependencies between them
  - hence, a dependence analysis of a given algorithm is necessary
  - example

```
for_all_processes i ← 0 to N do
    a[i] ← i + 1
od
```

  - what about the following code

```
for_all_processes i ← 1 to N do
    x ← i – 2*i + i*i
    a[i] ← a[x]
od
```

  - as it is not always obvious, an algorithmic way of recognising dependencies (via the compiler, e.g.) would preferable

Terms and Definitions

- dependence analysis (cont'd)
  - BERNSTEIN (1966) established a set of conditions, sufficient for determining whether two processes can be executed in parallel
  - definitions
    - I_i (input): set of memory locations read by process P_i
    - O_i (output): set of memory locations written by process P_i
  - BERNSTEIN's conditions

$$I_1 \cap O_2 = \varnothing \qquad I_2 \cap O_1 = \varnothing \qquad O_1 \cap O_2 = \varnothing$$

  - example

P_1: $a \leftarrow x + y$ $\qquad\qquad$ P_2: $b \leftarrow x + z$

$I_1 = \{x, y\}$, $O_1 = \{a\}$, $I_2 = \{x, z\}$, $O_2 = \{b\}$ ➔ all conditions fulfilled

Terms and Definitions

- dependence analysis (cont'd)
  - further example

P_1: $a \leftarrow x + y$ $\qquad\qquad$ P_2: $b \leftarrow a + b$

$I_1 = \{x, y\}$, $O_1 = \{a\}$, $I_2 = \{a, b\}$, $O_2 = \{b\}$ ➔ $\boldsymbol{I_2 \cap O_1 \neq \varnothing}$

  - BERNSTEIN's conditions help to identify instruction-level parallelism or coarser parallelism (loops, e.g.)
  - hence, sometimes dependencies within loops can be solved
  - example: two loops with dependencies – which to be solved

loop *A*:

```
for i ← 2 to 100 do
    a[i] ← a[i–1] + 4
od
```

loop *B*:

```
for i ← 2 to 100 do
    a[i] ← a[i–2] + 4
od
```


Terms and Definitions

- dependence analysis (cont'd)
  - expansion of loop *B*

```
a[2] ← a[0] + 4        a[3] ← a[1] + 4
a[4] ← a[2] + 4        a[5] ← a[3] + 4
a[6] ← a[4] + 4        a[7] ← a[5] + 4
   ⋮                      ⋮
```

  - hence, *a*[3] can only be computed after *a*[1], *a*[4] after *a*[2], … ➔ computation can be split into two independent loops

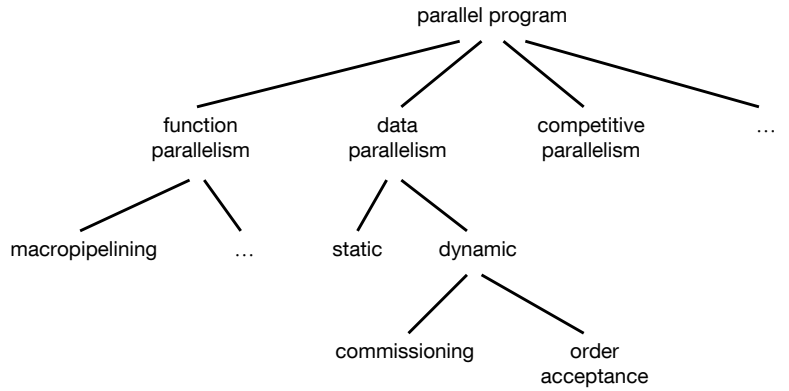
```
a[0] ← ...
for i ← 1 to 50 do
    j ← 2*i
    a[j] ← a[j–2] + 4
od
```

```
a[1] ← ...
for i ← 1 to 50 do
    j ← 2*i + 1
    a[j] ← a[j–2] + 4
od
```

  - many other techniques for recognising / creating parallelism exist (see also Chapter 4: Dependence Analysis)

Terms and Definitions

- structures of parallel programs
  - examples of structures



Terms and Definitions

- function parallelism
  - parallel execution (on different processors) of components such as functions, procedures, or blocks of instructions
  - drawback
    - separate program for each processor necessary
    - limited degree of parallelism ➔ limited scalability
  - macropipelining for data transfer between single components
    - overlapping parallelism similar to pipelining in processors
    - one component (producer) hands its processed data to the next one (consumer) ➔ stream of results
    - components should be of same complexity (➔ idle times)
    - data transfer can either be synchronous (all components communicate simultaneously) or asynchronous (buffered)

Terms and Definitions

- data parallelism
  - parallel execution of same instructions (functions or even programs) on different parts of the data (SIMD)
  - advantages
    - only one program for all processors necessary
    - in most cases ideal scalability
  - drawback: explicit distribution of data necessary (MesMS)
  - structuring of data parallel programs
    - *static*: compiler decides about parallel and sequential processing of concurrent parts
    - *dynamic*: decision about parallel processing at run time, i.e. dynamic structure allows for load balancing (at the expenses of higher organisation / synchronisation costs)

Terms and Definitions

- competitive parallelism
  - parallel execution of different processes (based on different algorithms or strategies) all solving the same problem
  - advantages
    - as soon as first process found the solution, computations of all subsequent processes are allowed to stop
    - on average, superlinear speed-up possible
  - drawback
    - lots of different programs necessary
  - nevertheless, rare case of parallel programs
  - examples: sorting algorithms, theorem proving within computational semantics

Terms and Definitions

- parallel programming languages
  - explicit parallelism
    - parallel programming interfaces
      - extension of sequential languages (C, Fortran, e.g.) by additional parallel language constructs
      - implementation via procedure calls from respective libraries
      - example: MPI, PVM, Linda
    - parallel programming environments
      - parallel programming interface plus additional tools such as compiler, libraries, debugger, …
      - most (machine dependent) environments come along with a parallel computer
      - example: MPICH

Terms and Definitions

- parallel programming languages (cont'd)
  - implicit parallelism
    - mapping of programs (written in a sequential language) to the parallel computer via compiler directives
    - primarily for the parallelisation of loops
    - only minor modifications of source code necessary
    - level of parallelism
      - block level for parallelising compilers (➔ threads)
      - instruction / sub-instruction level for vectorising compilers
    - example: OpenMP (parallelising), Intel compiler (vectorising)

Overview

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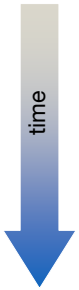
Process Interaction for MemMS

- problem: ATM race condition with two withdraw threads



thread 1	thread 2	balance
(withdraw $50)	(withdraw $50)	
read balance: $125		$ 125
	read balance: $125	$ 125
	set balance: $(125–50)	$ 75
set balance: $(125–50)		$ 75
give out cash: $50		$ 75
	give out cash: $50	$ 75

Process Interaction for MemMS

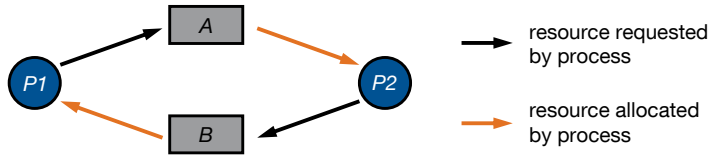
- principles
  - processes depend from each other if they have to be executed in a certain order; this can have two reasons
    - *cooperation*: processes execute parts of a common task
      - producer/consumer: one process generates data to be processed by another one
      - client/server: same as above, but second process also returns some data (result of a computation, e.g.)
    - *competition*: activities of one process hinder other processes
  - synchronisation: management of cooperation / competition of processes ➔ ordering of processes' activities
  - communication: data exchange among processes
  - MemMS: realised via shared variables with read / write access

Process Interaction for MemMS

- synchronisation
  - two types of synchronisation can be distinguished
    - *unilateral*: if activity *A2* depends on the results of activity *A1* then *A1* has to be executed before *A2* (i.e. *A2* has to wait until *A1* finishes); synchronisation does not affect *A1*
    - *multilateral*: order of execution of *A1* and *A2* does not matter, but *A1* and *A2* are not allowed to be executed in parallel (due to write / write or read / write conflicts, e.g.)
  - activities affected by multilateral synchronisation are *mutual exclusive*, i.e. they cannot be executed in parallel and act to each other atomically (no activity can interrupt another one)
  - instructions requiring mutual exclusion are called *critical sections*
  - synchronisation might lead to deadlocks (mutual blocking) or lockout ("*starvation*") of processes, i.e. indefinable long delay

Process Interaction for MemMS

- synchronisation (cont'd)
  - necessary and sufficient constraints for deadlocks
    - resources are only exclusively useable
    - resources cannot be withdrawn from a process
    - processes do not release assigned resources while waiting for the allocation of other resources
    - there exists a cyclic chain of processes that use at least one resource needed by the next processes within the chain



Process Interaction for MemMS

- synchronisation (cont'd)
  - possibilities to handle deadlocks
    - deadlock detection
      - techniques to detect deadlocks (identification of cycles in waiting graphs, e.g.) and measures to eliminate them (rollback, e.g.)
    - deadlock avoidance
      - by rules: paying attention that at least one of the four constraints for deadlocks is not fulfilled
      - by requirements analysis: analysing future resource allocations of processes and forbidding states that could lead to deadlocks (HABERMANN's / banker's algorithm well known from OS, e.g.)

Process Interaction for MemMS

- methods of synchronisation
  - lock variable / mutex
  - semaphore
  - monitor
  - barrier

Process Interaction for MemMS

- lock variable / mutex
  - used to control the access to critical sections
  - when entering a critical section a process
    - has to wait until the respective lock is open, then enters and closes the lock, thus no other process can follow
    - opens the lock and leaves when finished
    - lock / unlock have to be executed from the same process
  - lock variables are abstract data types consisting of
    - a boolean variable of type mutex
    - at least two functions lock and unlock
    - further functions (Pthreads): init, destroy, trylock, …
  - function lock consists of two operations “test” and “set” which together form a non interruptible (i.e. atomic) activity

Process Interaction for MemMS

- semaphore
  - abstract data type consisting of
    - nonnegative variable of type integer (semaphore counter)
    - two atomic operations *P (“passeeren”)* and *V (“vrijgeven”)*
  - after initialisation of semaphore S the counter can only be manipulated with the operations $P(S)$ and $V(S)$
    - $P(S)$: if $S > 0$ then $S \leftarrow S - 1$
else the processes executing $P(S)$ will be suspended
    - $V(S)$: $S \leftarrow S + 1$
  - after a *V*-operation any suspended process is reactivated (busy waiting); alternatives: always next process in queue
  - *binary semaphore*: has only values “0” and “1”
  - *general semaphore*: has any nonnegative number

Process Interaction for MemMS

- semaphore (cont’d)
  - initial value of semaphore counter defines the maximum amount of processes that can enter a critical section simultaneously
  - critical section enclosed by operations *P* and *V*

```
# mutual exclusion
(binary) semaphore s; s ← 1
execute p1() and p2() in parallel

begin procedure p1()
    while (true) do
        P(s)
        enter_crit_section()
        V(s)
    od
end
```

```
begin procedure p2()
    while (true) do
        P(s)
        enter_crit_section()
        V(s)
    od
end
```


Process Interaction for MemMS

- semaphore (cont’d)
  - consumer/producer-problem: semaphore indicates difference between produced and consumed elements
  - assumption: unlimited buffer, atomic operations store and remove

```
# consumer/producer
(general) semaphore s; s ← 0
execute producer() and consumer() in parallel

begin procedure producer()
    while (true) do
        produce X
        store X
        V(s)
    od
end
```

```
begin procedure consumer()
    while (true) do
        P(s)
        remove X
        consume X
    od
end
```

Process Interaction for MemMS

- monitor
  - semaphores solve synchronisation on a very low level ➔ already one wrong semaphore operation might cause the breakdown of the entire system
  - better: synchronisation on a higher level with monitors
    - abstract data type with implicit synchronisation mechanism, i.e. implementation details (such as access to shared data or mutual exclusion) are hidden from the user
    - all access operations are mutual exclusive, thus all resources (controlled by the monitor) are only exclusively useable
  - monitors consist of
    - several monitor variables and monitor procedures
    - a monitor body (instructions executed after program start for initialisation of the monitor variables)

Process Interaction for MemMS

- monitor (cont'd)
  - only access to monitor-bound variables via monitor procedures, direct access from outside the monitor is not possible
  - only one process can enter a monitor at each point in time, all others are suspended and have to wait outside the monitor
  - synchronisation via condition variables (based on mutex)
    - *wait(c)*: calling process is blocked and appended to an internal queue of processes also blocked due to condition *c*
    - *signal(c)*: if queue for condition *c* is not empty, the process at the queue's head is reactivated (and also preferred to processes waiting outside for entering the monitor)
  - condition variables are monitor-bound and only accessible via operations wait and signal (➔ no manipulation from outside)

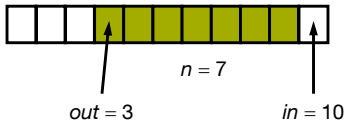
Process Interaction for MemMS

- monitor (cont'd)
  - consumer/producer-problem with limited buffer (1)

```
# consumer/producer
const size ← ...

monitor limitedbuffer
    buffer[size] of integer
    in, out: integer
    n: integer
    notempty, notfull: condition

begin procedure store(X)
    if n = size then wait(notfull) fi
    buffer[in] ← X; in ← in + 1
    if in = size then in ← 0 fi
    n ← n + 1
    signal(notempty)
end
```



Process Interaction for MemMS

- monitor (cont'd)
  - consumer/producer-problem with limited buffer (2)

```
begin procedure remove(X)
    if n = 0 then wait(notempty) fi
    X ← buffer[out]; out ← out + 1
    if out = size then out ← 0 fi
    n ← n – 1
    signal(notfull)
end

monitor body: in ← 0; out ← 0; n ← 0

begin procedure producer()
    while (true) do
        produce X
        store(X)
    od
end

begin procedure consumer()
    while (true) do
        remove(X)
        consume X
    od
end
```

Process Interaction for MemMS

- monitor (cont'd)
  - some remarks
    - compared to semaphores, after once correctly programmed monitors cannot be disturbed by adding further processes
    - semaphores can be implemented via monitors and vice versa
    - nowadays, multithreaded OS use lock variables and condition variables instead of monitors
    - nevertheless, monitors are used within Java for synchronisation of threads

Process Interaction for MemMS

- barrier
  - synchronisation point for several processes, i.e. each process has to wait until the last one also arrived
  - initialisation of counter C before usage with the amount of processes that should wait (init-barrier operation)
  - each process executes a wait-barrier operation
    - counter C is decremented by one
    - process is suspended if $C > 0$, otherwise all processes are reactivated and the counter C is set back to the initial value
  - useful for setting all processes (after independent processing steps) into the same state and for debugging purposes

Process Interaction for MemMS

- simple case study

"Program testing can be used to show the presence of bugs, but never to show their absence."

E.W. Dijkstra

Process Interaction for MemMS

- simple case study (cont'd)
  - test case: reader-writer-problem
  - to be examined
    - *deadlock*: program will never deadlock
    - *mutual exclusion*: resource is never used by both processes at same time
    - *liveness*: resource will eventually be used by any process
  - status variables
    - x, pc_0, pc_1
    - hence, 32 states possible (but 12 states not reachable)

```
                boolean x ← 0

                proc rw0 {
                  while (true) {
0:                  x ← 0
1:                  sync ( )
2:                  if (x = 0)
3:                    use_resource
                  }
                }
                proc rw1 {
                  while (true) {
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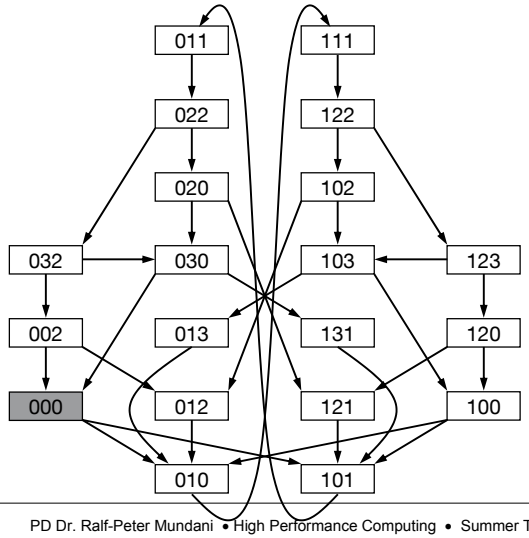
                }
```

Process Interaction for MemMS

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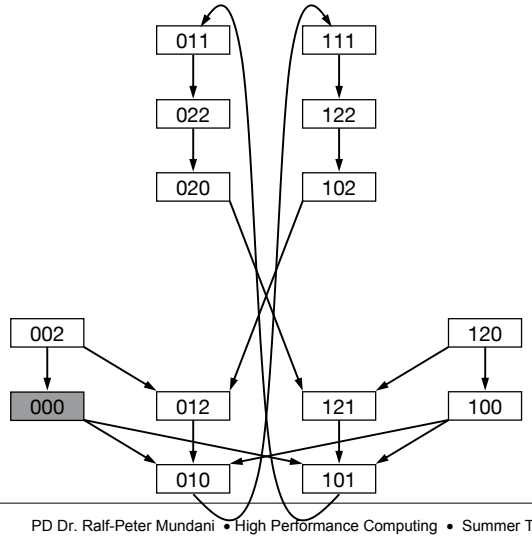
Process Interaction for MemMS

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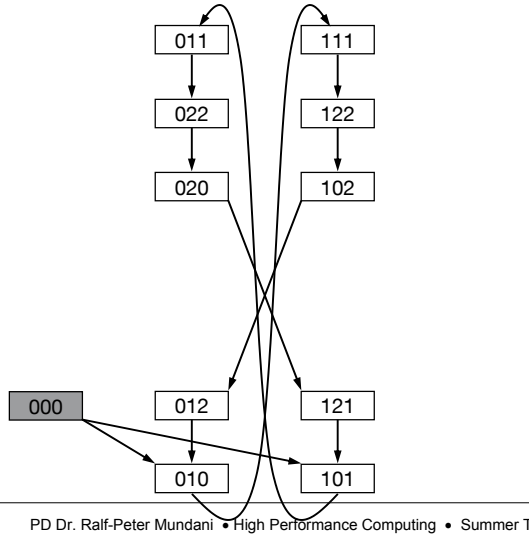


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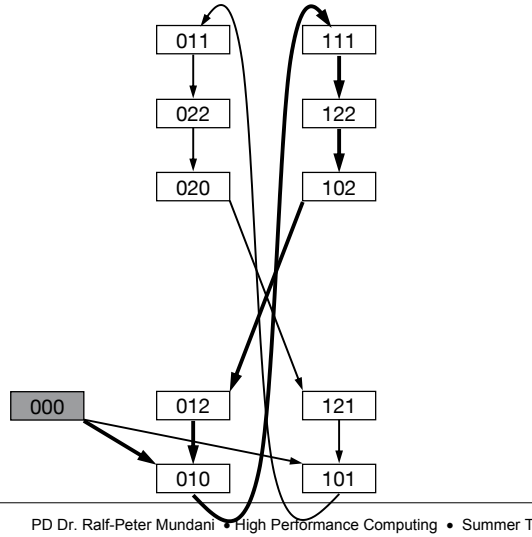


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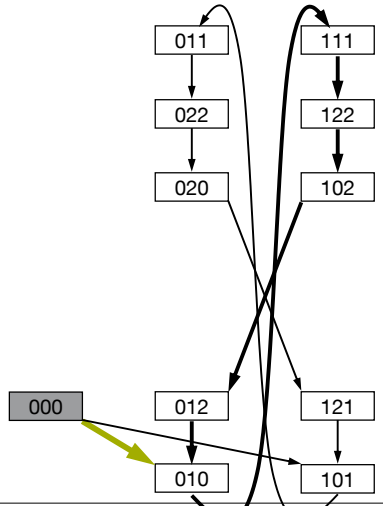


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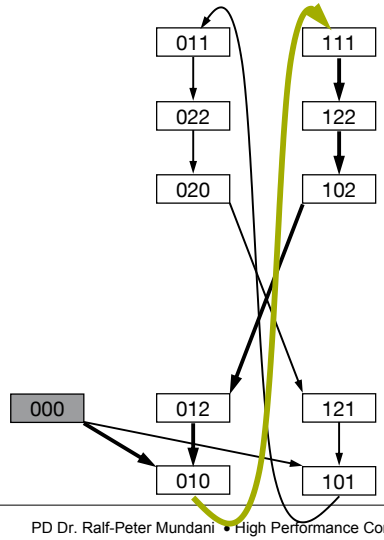


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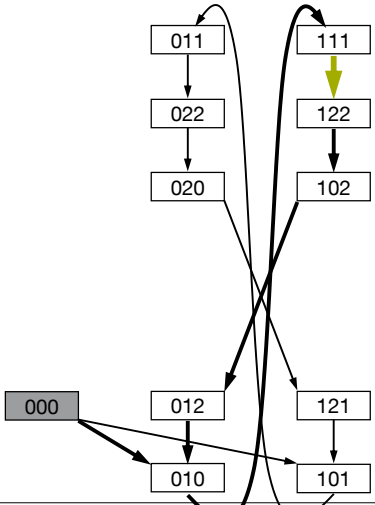


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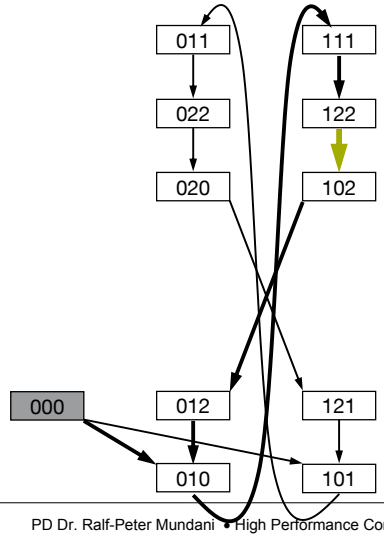


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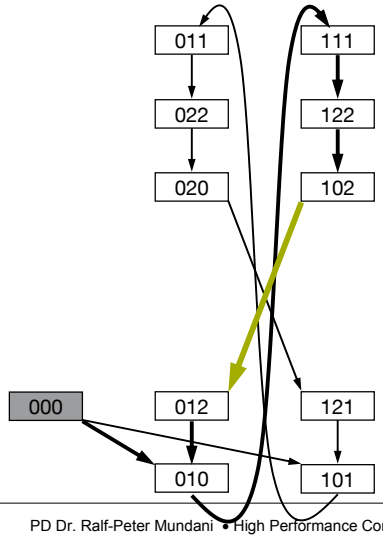


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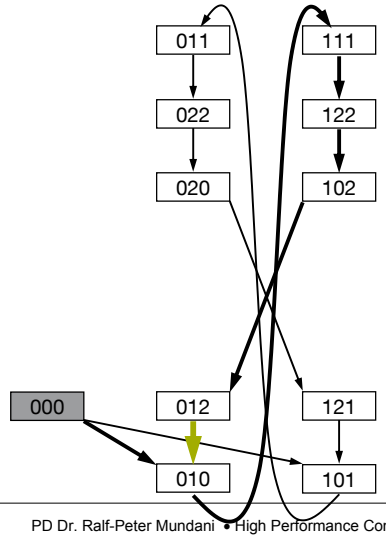


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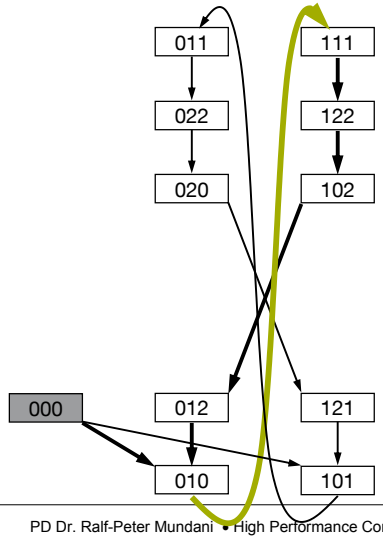


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   while (true) {
0:    x ← 1
1:    sync ( )
2:    if (x = 1)
3:       use_resource
   }
}
```


Overview

- terms and definitions
- process interaction for MemMS
- process interaction for MesMS
- example of a parallel program
- load balancing
- state-of-the-art LB strategies

Process Interaction for MesMS

- message passing paradigm
  - no shared memory for synchronisation and communication
  - hence, transfer mechanism for information interchange necessary
  - message passing
    - messages: data units transferred between processes
    - send / receive operations instead of read / write operations
  - implicit (sequential) order during send / receive-stage
    - a message can only be received after a prior send
    - communication via message passing (independent from the transferred data) leads to an implicit synchronisation
    - synchronisation due to availability / unavailability of messages

Process Interaction for MesMS

- messages
  - created whenever a process performs a send
  - necessary information to be provided from the sender
    - destination (process, node, communication channel, e.g.)
    - unique identifier of message (number, e.g.)
    - memory (so called send buffer) containing the data to be transferred with the message
    - data type and amount of elements within the send buffer
  - data type and amount of elements have to match for the receiver, otherwise a correct interpretation of the data is in doubt

Process Interaction for MesMS

- sending messages
  - send operations can be
    - *synchronous / asynchronous*: a process performing a send is either blocked (synchronous) or not (asynchronous) until a respective receive operation is executed
    - *buffered / unbuffered*: a send operation may first copy the data from the send buffer to a system buffer (buffered) or directly perform the transfer (unbuffered ➔ faster, but risk of overwriting the send buffer due to parallel execution of transfer (NIC) and next send operation (CPU) possible)
    - *blocking / non-blocking*: a send operation can either be blocked until the send buffer has been emptied (blocking) or immediately give control to the next instruction of the sending process (non-blocking ➔ risk of overwriting send buffer)

Process Interaction for MesMS

- receiving messages
  - a process has to specify which message to receive (via message identifier or wildcard) and where to store the data (so called receive buffer)
  - receive operations can be
    - *destructive / non-destructive*: a receive operation copies the data transferred with the message into the receive buffer and either destroys the message (destructive) or keeps it for later usage (non-destructive)
    - *synchronous / asynchronous*: a process performing a receive is either blocked until a message has arrived (synchronous) or not (asynchronous), thus it can continue with its execution and check again at a later point in time

Process Interaction for MesMS

- addressing modes
  - different addressing modes can be distinguished
    - *direct naming*: process identifiers are used for sender and receiver ➔ identifiers have to be known during development
    - *mailbox*: global memory where processes can store (send) and remove (receive) messages (used in the Distributed Execution and Communication Kernel (DECK), e.g.)
    - *port*: a port is bound to one process and can be used in one direction only, i.e. either for sending or receiving messages
    - *connection / channel*: for the usage of ports the setup of connections or channels is required, i.e. the connection of a send port of one process with the receive port of another process ➔ data written from the sender to its port can be read from the receiver on the other port

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Example of a Parallel Program

- labyrinth
  - given: map of some labyrinth that contains
    - one entrance
    - one exit
    - no cycles

  - task: determine if there exists a way from the entrance through the labyrinth to the exit (not the way itself)

Example of a Parallel Program

- labyrinth (cont'd)
  - map stored as matrix of size $M \times N$

```
0.0000000000000000000000000000000000000000000000000000000000000000000000
0.0000000000000000000....0000000000000000.....0000000000000
0......00000000....000.0000000..........00.000......00000000
000000.00000....00.000.0000000.00000000.00.0000.000.......00
000000.......00000.000.000.....00000000....0000.000000000.00
000000.00000000000.000.000.00000...0000.0000000.000000000.00
000000.00000000000.000.000...0000.00....0000....000000000.00
000000.00000000000.000.00000.0000.00.0000000000000000000.00
000000.000.........000.00000.0000....0000000.............00
000000.000.00000000000.00000.0000000.0000000000000.000000000
000000.000.................0000000.000000.000000.000000000
00.....0000000000000000000000000000.000000.000000.000000000
000000.00000000000000000000000000.........000000.000000000
000000.....................0000000000.000000.000000.000000000
0000000000000000000000000000000000....000000.000000.000000000
0000000000000000000000000000000000000000000000000.000000000
```

Example of a Parallel Program

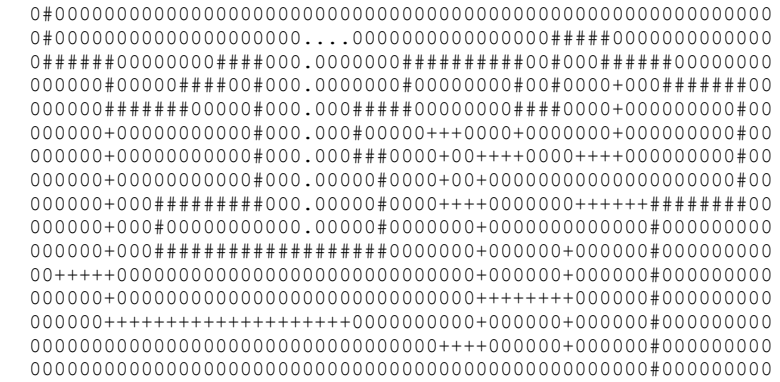
- labyrinth (cont'd)
  - sequential algorithm (1)

```
position ← entrance

while (true) do
    switch (position) do
        case 'crossroads':  position ← TURN_RIGHT()
        case 'dead end':    position ← TURN_BACK()
        case 'exit':        halt(OKAY)
        case 'entrance':    halt(ERROR)
    od
od
```


Example of a Parallel Program

- labyrinth (cont'd)
  - sequential algorithm (2)



Example of a Parallel Program

- labyrinth (cont'd)
  - parallelisation
    - basic questions
      - which parts to be parallelised
      - which structure: function or data parallelism
      - which model: shared or distributed memory

Example of a Parallel Program

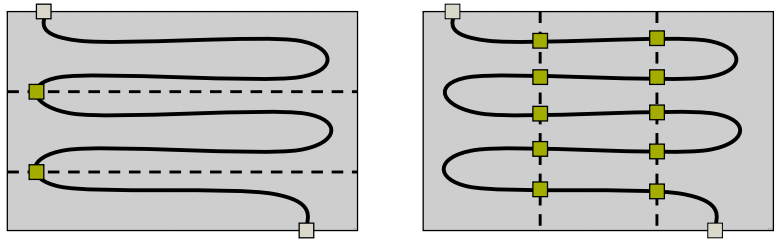
- labyrinth (cont'd)
  - variant A: *function parallelism*
    - start new processes at crossroads
    - terminate processes at dead ends
    - halt in case
      - one process has reached the exit
      - all processes have been terminated
    - questions
      - shared or distributed memory
      - total number of processes ➔ queue of tasks
      - drawbacks

Example of a Parallel Program

- labyrinth (cont'd)
  - variant B: *competitive parallelism*
    - start *N* processes following *N* different algorithms
    - first process reaching exit or entrance tells others to stop
    - possible algorithms
      - always go left instead of going right
      - start from the exit and try to reach the entrance
      - randomly walk around and remember all paths that have already been examined
      - ...
    - questions
      - shared or distributed memory
      - drawbacks

Example of a Parallel Program

- labyrinth (cont'd)
  - variant C: *data parallelism* (1)
    - naïve approach
      - cut matrix into *N* parts
      - solve corresponding subproblems and check if there exist matching entrance-exit pairs ➔ communication overhead



Example of a Parallel Program

- labyrinth (cont'd)
  - variant C: *data parallelism* (2)
    - "better" approach
      - cut matrix *A* into parts A_i and distribute among processes
      - flood fill each part A_i with different "colors"
      - collect boundaries of all A_i
      - check if (color) transition from entrance to exit is possible
    - questions
      - shared or distributed memory
      - drawbacks

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Load Balancing

- motivation
  - central issue: fairly distribution of computations across all processors / nodes in order to optimise
    - run time (user's point of view)
    - system load (computing centre's point of view)
  - problem
    - amount of work is often not known prior to execution
    - load situation changes permanently (adaptive mesh refinement within numerical simulations, I/O, searches, …)
    - different processor speeds (heterogeneous systems, e.g.)
    - different latencies for communication (grid computing, e.g.)
  - objective: load distribution or load balancing strategies

Load Balancing

- static approaches
  - to be applied before the execution of any process (in contrast to dynamic load balancing to be applied during the execution)
  - usually referred to as mapping problem or scheduling problem
  - potential static load-balancing techniques
    - *round robin*: assigning tasks (more general formulation than work to cover both data and function parallelism) in sequential order to processes, coming back to the first when all processes have been given a task
    - *randomised*: selecting processes at random to assign tasks
    - *recursive bisection*: recursive division into smaller subproblems of equal computational effort with less communication costs

Load Balancing

- dynamic approaches
  - division of tasks dependent upon the execution of parts of the program as they are being executed ➔ entails additional overhead (to be kept as small as possible, else bureaucracy wins)
  - assignment of tasks to processes can be classified as
    - *centralised*
      - tasks are handed out from a centralised location
      - within a master-slave structure one dedicated master process directly controls each of a set of slave processes
    - *decentralised*
      - tasks are passed between arbitrary processes
      - worker processes interact among themselves ➔ a worker process may receive / send tasks from / to others

Load Balancing

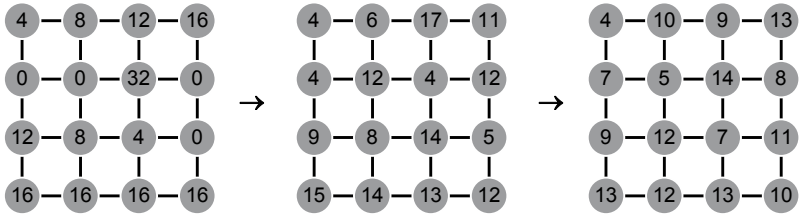
- diffusion model (a. k. a first order scheme)
  - analogy to physical processes in nature (salt or ink in water, e.g.)
  - original algorithm introduced by CYBENKO (1989) for static network topologies, meanwhile it has been often studied and derived (second order scheme, dynamic network topologies, e.g.)
  - *idea*: a process P_i balances its load simultaneously with all its neighbours $N(i)$ ➔ ratio α_{ij} of the load difference between process P_i and P_j is swapped between them according to

$$w_i^{(t+1)} = w_i^{(t)} - \sum_{j \in N(i)} \alpha_{ij} \cdot (w_i^{(t)} - w_j^{(t)}), \quad 1 \le i \le p, \qquad -1 < \alpha_{ij} < 1$$

    where $w_i^{(t)}$ is the workload done by process P_i at time t
  - various methods to be found that determine parameter α_{ij} such as
    - optimal choice: needs global knowledge of the network
    - BOILLAT choice: needs only local knowledge of the neighbours

Load Balancing

- diffusion model (cont'd)
  - update of workload can be done
    a) after all balancing factors have been computed (JACOBI-like)
    b) during computation of balancing factors (GAUSS-SEIDEL-like)
  - example: first two iteration steps with $\alpha = 0.25$ for JACOBI method



initial setup ($t = 0$) first step ($t = 1$) second step ($t = 2$)

Load Balancing

- bidding (economic model)
  - analogy to mechanisms of price fixing in markets
  - idea
    - process (with high workload) advertises tasks to its neighbours
    - neighbours submit their free resources as bid
    - process with highest bid (i.e. largest free resources) wins
  - remarks
    - maybe several rounds of bidding necessary ➔ successively extending the range of bidders
    - in case of sudden workload peaks, a process might reject the purchased tasks
    - processes with free resources are still allowed to ask for tasks
  - drawback: quite complex analysis of this model

Load Balancing

- precalculation of the load
  - all strategies so far are based on local information only
  - hence, load balancing is often quite expensive since (from a global point of view) balancing steps not always lead to a better load distribution among the processors
  - idea
    - global determination of the workload at program start or at certain points in time
    - global determination of an appropriate load distribution
    - workload transfer with less communication
  - developed and used for hierarchical network topologies ➔ workload recording and load balancing between child and parent nodes

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State-of-the-art LB Strategies

- space-filling curves
  - origin of the idea: analysis and topology (“topological monsters”)
  - nice example of a construct from pure mathematics that gets practical relevance only decades later
  - definition of a space filling curve (SFC)
    - curve: image of a continuous mapping f: $[0,1] \rightarrow [0,1]^D$
    - SFC: continuous, surjective mapping f: $[0,1] \rightarrow [0,1]^D$ that covers an area (with a JORDAN content) greater than zero
  - prominent representatives
    - HILBERT’s SFC (1891): most famous SFC
    - PEANO’s SFC (1890): oldest SFC
    - LEBESGUE’s SFC: most important SFC for computer science
  - further reading: M. Bader, *Space-Filling Curves*, Springer (2012)

State-of-the-art LB Strategies

- HILBERT’s space filling curve
  - for reasons of simplicity only in 2D ➔ f: $I = [0,1] \rightarrow [0,1]^2 = Q$
  - construction of SFC follows the geometric conception

    If I can be mapped onto Q in the space filling sense, then each of the four congruent subintervals of I can be mapped to one of the four quadrants of Q in the space filling sense, too.

  - recursive application of above preserves
    - *neighbourhood relations*: neighbouring subintervals in I are mapped onto neighbouring subsquares of Q
    - *subset relations (inclusion)*: from $I_1 \subseteq I_2$ follows $f(I_1) \subseteq f(I_2)$
  - border case: HILBERT’s SFC

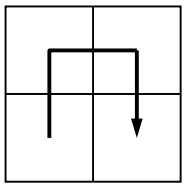
State-of-the-art LB Strategies

- HILBERT’s space filling curve (cont’d)
  - correspondence of nested intervals in I and nested squares in Q provides pairs of points in I with corresponding image points in Q
  - of course, the iterative steps in this generation process are of practical relevance, not the border case itself

    starting with a generator or “Leitmotiv” that defines the order in which the subsquares are visited

    applying generator in each subsquare (with appropriate similarity transformations if necessary)

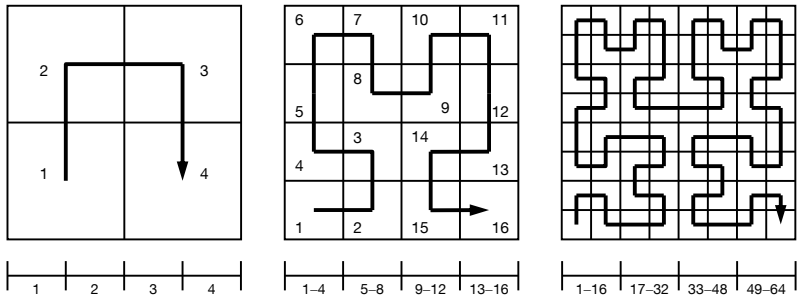
    connecting the open ends



generator for HILBERT’s SFC

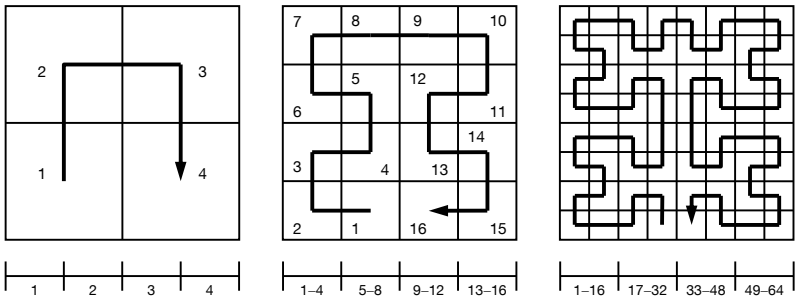
State-of-the-art LB Strategies

- HILBERT’s space filling curve (cont’d)
  - classical version of HILBERT



State-of-the-art LB Strategies

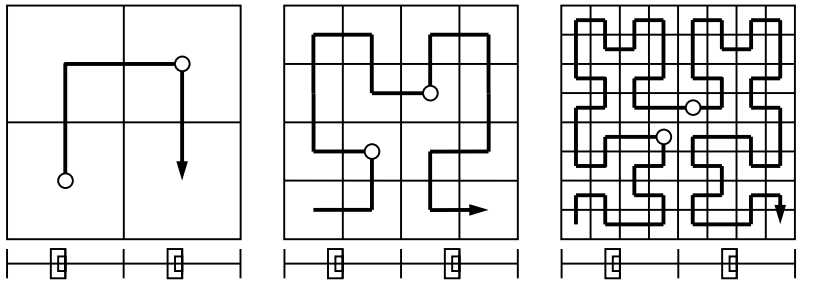
- HILBERT's space filling curve (cont'd)
  - variant of MOORE



  - modulo symmetry, these are the only two possibilities

State-of-the-art LB Strategies

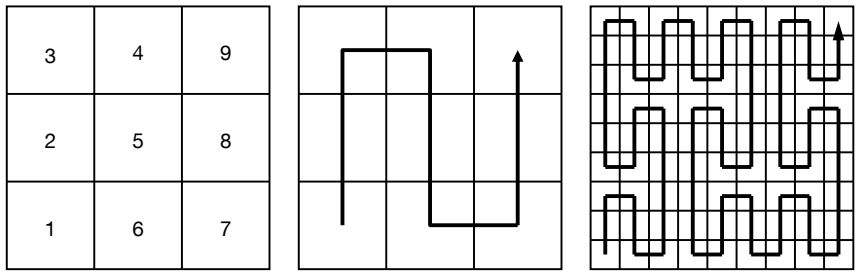
- HILBERT's space filling curve (cont'd)
  - all iterations are injective, but HILBERT's SFC itself is not injective (there are image points with more than one original point in *I*)



note: there exists a bijective mapping between two finite-dimensional smooth manifolds (CANTOR, 1878), but it cannot be both bijective and continuous (NETTO, 1879)

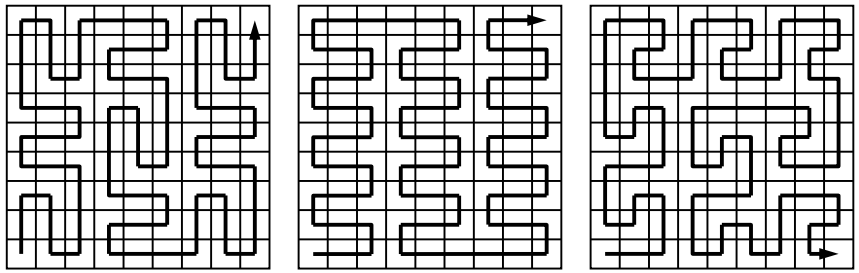
State-of-the-art LB Strategies

- PEANO's space filling curve
  - ancestor of all SFCs
  - subdivision of *I* and *Q* into nine congruent subdomains
  - definition of a generator, again, defines the order of visit



State-of-the-art LB Strategies

- PEANO's space filling curve (cont'd)
  - now, there are (modulo symmetry) 273 different possibilities to recursively apply the generator preserving neighbourhood and inclusion



serpentine type (left and centre) and meander type (right)

State-of-the-art LB Strategies

- LEBESGUE's space filling curve
  - definition of LEBESGUES's SFC by the CANTOR set
  - CANTOR set *C*: repeatedly deleting the open middle thirds of [0,1]

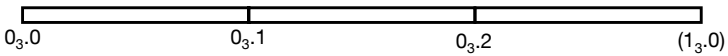
  - *C* is defined as set of points not excluded, hence the remaining interval can be computed by the total length removed

$$\sum_{N=0}^{\infty} \frac{2^N}{3^{N+1}} = \frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \cdots = \frac{1}{3} \cdot \left(\frac{1}{1 - \frac{2}{3}} \right) = 1$$

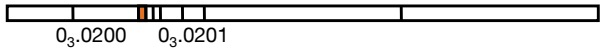
  - the proportion of the remaining interval seems to be $1 - 1 = 0$, but in fact *C* has the same cardinality as the unit interval [0,1] (!)

State-of-the-art LB Strategies

- LEBESGUE's space filling curve (cont'd)
  - nested intervals of *C* to be represented by ternary numbers of the form $0_3.w_1w_2w_3\ldots$ with $w_i \in \{0, 1, 2\}$



  - example: parameter $T = 2/9$ can be written as [0,1], $[0_3.0,0_3.1]$, $[0_3.02,0_3.10]$, $[0_3.020,0_3.021]$, $[0_3.0200,0_3.0201]$, …



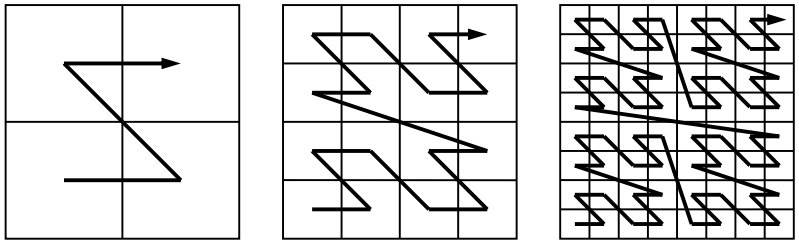
  - since all interval borders can be written in two different ways ($1_3.0$ or $0_3.222\ldots$, e.g.) and the middle third ($[0_3.1,0_3.2]$, e.g.) is repeatedly deleted, the CANTOR set only contains ternary numbers that consist of "0" and "2"

State-of-the-art LB Strategies

- LEBESGUE's space filling curve (cont'd)
  - when mapping *C* to $[0,1]^2$ according to

$$f : \left(\frac{0_3.w_1w_2w_3w_4\ldots}{2} \right) = \begin{pmatrix} 0_2.x_1x_3\ldots \\ 0_2.y_2y_4\ldots \end{pmatrix}$$

    and connecting the image points via linear interpolation, this results to LEBESGUE's SFC also referred to as "*Z*-order"



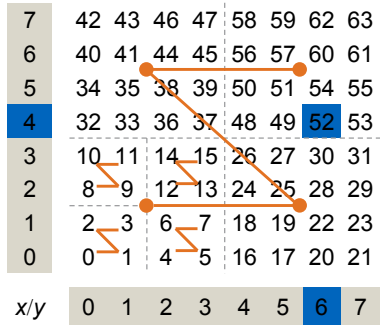
State-of-the-art LB Strategies

- LEBESGUE's space filling curve (cont'd)
  - *Z*-ordering is well-known from quadtrees and octrees when linearising a tree by a depth-first traversal (provides a common naming scheme for cells ➔ lexicographic or MORTON index)
  - bitwise interleaving of coordinate values (*x*, *y*) leads to *Z*-value
  - useful for multidimensional range searches, e.g.

x: 02.100 → 4

y: 02.110 → 6

Z: 02.110100 → 52



State-of-the-art LB Strategies

- applications
  - sequentialisation of multidimensional data to one dimension while preserving locality
    - data are stringed sequentially like pearls
    - neighbouring points in the unit interval [0,1] have neighbouring images in $[0,1]^D$
  - important applications such as
    - efficient multidimensional range searches in databases (Oracle, e.g.)
    - multi-particle or *N*-body problems
    - adaptive grid refinement for partial differential equations
    - dynamic load balancing

State-of-the-art LB Strategies

- load balancing
  - idea
    1) assign points q_i of some iteration of an SFC (i.e. $q_i \in Q$) to points s_i in the *D*-dimensional space
    2) linearly order points q_i according to SFC's original points $p_i \in I$
    3) simple partitioning based on this sequential order of points p_i (while preserving locality) to processors possible