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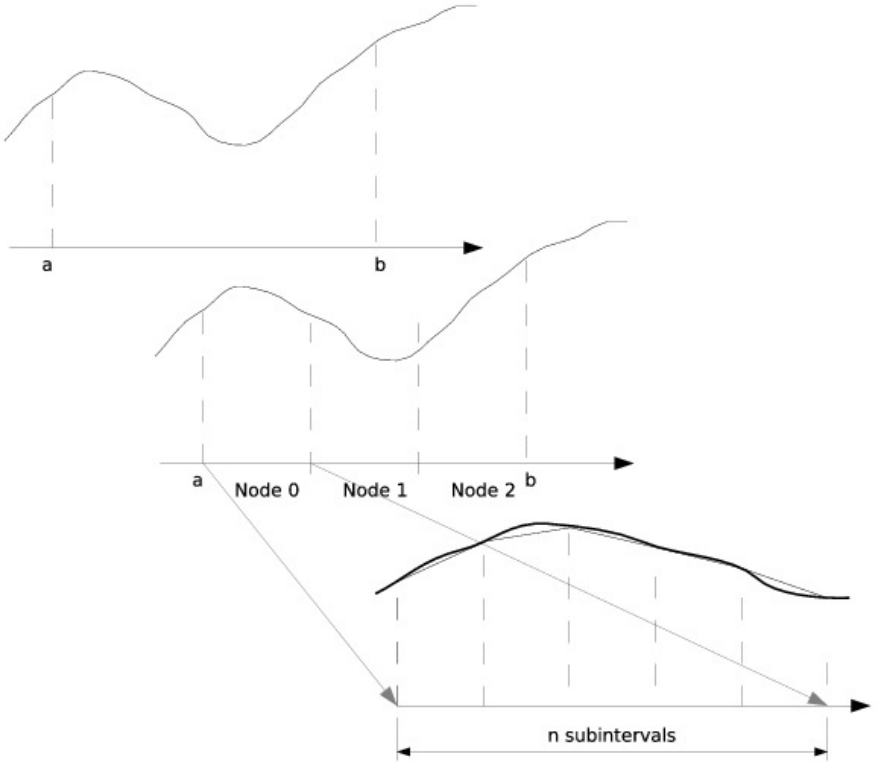
Parallel Numerics

Exercise 2: Numerical Integration & P2P Communication I

1) Numerical Integration



We are to calculate the definite integral of a function $f(x)$ in an interval $[a, b]$ using the trapezoidal rule. Before, we split up the overall interval $[a, b]$ into `#proc` equidistant subintervals $[a_k, b_k]$, $k \in \{0, 1, \ldots, \#proc - 1\}$. `#proc` is the number of nodes available on a parallel computer. Afterwards, we divide the interval $[a_k, b_k]$ into n intervals with length $h = (b_k - a_k)/n$

and approximate the integral by a sum of trapezoids:

$$\begin{aligned}\int_{a_k}^{b_k} f(x)\,dx &\approx \sum_{i=1}^{n} \frac{h}{2}\Big(f(a_k+(i-1)h)+f(a_k+ih)\Big)\\ &= \frac{h}{2}\Big(f(a_k)+f(b_k)\Big)+h\cdot\sum_{i=1}^{n-1} f(a_k+ih) \qquad (1)\end{aligned}$$

i) Implement the algorithm using the MPI send and receive operations.
See sourcecode to corresponding tutorial on webpage.

ii) Test the application for $f(x) = -3x^2 + 3$, $[a, b] = [-1, 1]$, $n = 2, 4, 8$, and different numbers of processors.
Test on your own with sourcecode from i).

iii) Test the application for $f(x) = x\sin(10x)$, $[a, b] = [0, 2\pi]$, $n = 2, 4, 8$ and different numbers of processors.

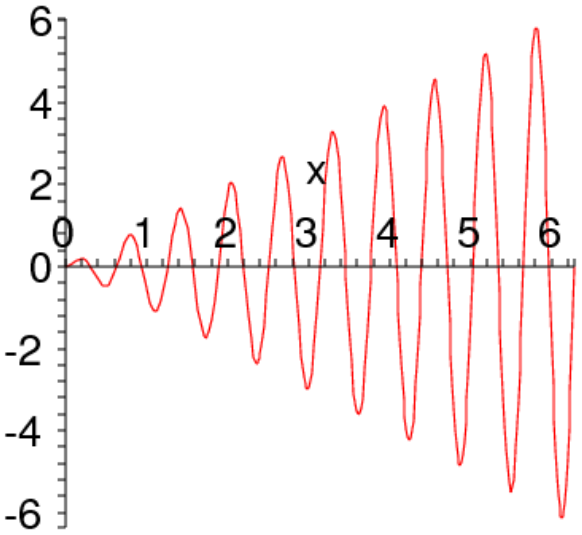


Figure 1: $f(x) = x\sin(10x)$.

Compute the analytical solution and compare it to the numerical one. What do you observe and why?
If function is not smooth a sufficient small mesh size h is required to reduce the computed numerical error. The numerical error is not a parallel phenomenon but relies more on where the boundaries of the subintervals (number of processes) were chosen. One possible remedy is adaptivity.

iv) Implement the following strategy for determining n on a node:

- n = 1
- Compute $F^{(1)}_{numerical}$ (see Formula 1)

- n = 2
- Compute $F^{(2)}_{numerical}$ (see Formula 1)
- n = 3
- Compute $F^{(3)}_{numerical}$ (see Formula 1)
- ...
- Quit if $F^{(k+1)}_{numerical} - F^{(k)}_{numerical} < \epsilon$

Test the algorithm for

$$f(x) = \begin{cases} 0 & for\ x \leq \frac{1}{2} \\ 1 & for\ x > \frac{1}{2} \end{cases} , \tag{2}$$

$[a, b] = [0, 1]$, $n = 2, 3, 4$ and different values for ϵ. Try to find out, how many iterations per node are performed. Is there a bottleneck?
Test on your own. For non-smooth functions the mesh size has to be sufficient small. This trivial domain splitting is overkill for functions with only few non-smooth regions like for instance a tight gaussian peak. Disadvantage: adaptive schemes make load balancing challenging.

v) Before the application terminates, each node sends its contribution to the numerical result to node 0. You have used MPI send and receive to implement this behaviour. Is there a technical term for such a communication scheme?
Reduce/Reduction.

2) MPI_Send and MPI_Recv

i) Write an operation sequence that receives four integers from arbitrary senders (use the constant `MPI_ANY_SOURCE` as source number for the `MPI_Recv` operation) using the message tag `0` for each of them. Store the results in four integers `b0,b1,b2` and `b3`.

```
int b0,b1,b2,b3;
MPI_Recv(&b0,1,MPI_INT,MPI_ANY_SOURCE,0,MPI_COMM_WORLD);
MPI_Recv(&b1,1,MPI_INT,MPI_ANY_SOURCE,0,MPI_COMM_WORLD);
MPI_Recv(&b2,1,MPI_INT,MPI_ANY_SOURCE,0,MPI_COMM_WORLD);
MPI_Recv(&b3,1,MPI_INT,MPI_ANY_SOURCE,0,MPI_COMM_WORLD);
```

ii) Consider the following code snippet

```
int a=0;
MPI_Send(&a, 1, MPI_INT, 2, 0, MPI_COMM_WORLD);
a=1;
MPI_Send(&a, 1, MPI_INT, 2, 0, MPI_COMM_WORLD);
```

running on node 0, and

```
int a=2;
MPI_Send(&a, 1, MPI_INT, 2, 0, MPI_COMM_WORLD);
a=3;
```

```
MPI_Send(&a, 1, MPI_INT, 2, 0, MPI_COMM_WORLD);
```

running on node 1. What are possible values for `b0,b1,b2` and `b3` when executing the four `MPI_Recv` operations from i) on node 2 ? Is there a unique solution? Are messages allowed to overtake?
The arrival pattern is not deterministic if ***MPI_ANY_SOURCE*** *is used. One receive stores exactly one message in a variable. Messages from the same sender are not allowed to overtake. Thus, the following possibilities on node 2 are:*

b0	0	0	0	2	2	2
b1	1	2	2	0	0	3
b2	2	1	3	1	3	0
b3	3	3	1	3	1	1

iii) Define the term *blocking* for MPI commands.
Operation is left if variable content (buffer) is set/send completely and can be reused again.

iv) Define the term *not synchronized* for MPI commands.
The standard MPI operations are not synchronized, i.e., there is no handshake semantics and it cannot be assured that the message is going to be received.

3) Predefined MPI datatypes

For every MPI send or receive command, one has to specify a datatype. Make yourself familiar with the datatypes offered by the MPI interface. To what C datatypes do they correspond?

MPI	C (32 bit)
CHAR	signed char
SHORT	signed short int
INT	signed int
LONG	signed long int
FLOAT	float
DOUBLE	double