

Parallel Numerics

Exercise 7: Sparse Matrix–Vector Multiplication & MPI Communicators

1 MPI Communicators

- a) Give examples for some network topologies (e.g. cartesian grids, hypercubes, tree). How are the standard mpi ranks mapped onto such topologies?
- b) Which topology fits perfectly to Cannon's algorithm?
- c) Besides a topology, a mpi communicator holds a group, a communication context (communicator specific tagging) and communicator specific buffers. Thus, e.g. collective operations act only for elements of the same communicator. Make yourself familiar with the communicator concept.
- d) Give an application domain for mpi communicators.

2 Red-Black Colouring

The idea of any colouring approach for an iterative algorithm is to assign evaluations to different computing steps (colours). Afterwards, these steps are executed sequentially.

- a) What are typical properties of elements of one colour?
- b) Derive a colouring scheme for a 5-point stencil on a cartesian two-dimensional grid.
- c) Why is bicolouring for a 9-point stencil on a cartesian two-dimensional grid not well-suited? Derive an improved colouring.

3 Sparse Matrix-Vector Multiplication

The solution of the matrix-vector product $A \cdot b$ with a sparse matrix $A \in \mathbb{R}^{n \times n}$ and $b \in \mathbb{R}^n$ should be calculated. Write different procedures for the calculation of the matrix-vector multiplication. Implement the storage schemes for matrix A that have been discussed in the lecture:

1. storage in coordinate form
2. compressed sparse row format (CSR)
3. compressed sparse column format (CSC)
4. CSR with extraction of the main diagonal entries
5. diagonal-wise storage (for band matrices)
6. rectangular, row-wise storage scheme
7. jagged diagonal form

Test your routines for large sparse matrices whose elements are randomly distributed as zeros or as non-zero values. Compare the computation time to the time needed by applying the full matrix storage scheme.