Technische Universität München
Institut für Informatik
Lehrstuhl für numerische Programmierung

Introduction to Scientific Computing

Collection of Exam Questions – ODE

Classification of questions:

- • regular question – could well be part of the exam;
- ● more difficult question;
- ∗ probably too difficult or extensive for the exam;
- ❍ side topic – probably not part of the exam.

The list contains only questions that were generated by the students during the lectures. Consequently, there is no guarantee that the list is complete or representative. However, the questions cover an extensive part of the lecture, and I want to thank everyone who contributed to it.

If a questions says "Given a specific ...", then make up your own example (together with values of parameters) or modify an example from the lectures. Questions here will only reflect the types of possible questions.

1 Modelling

- Define the following terms in the classification of models:
  - discrete vs. continuous
  - determinate vs. stochastic
  - single vs. multivariable (dimensional)
- Classify a given (population) model according to this classification.
- Name some difference and possible advantages/disadvantages between discrete population models and ODE models.

- Where do the Fibonacci numbers come from?
- ● What is the difference between a discretised model and a discrete model?
- ❍ Explain the "scientific process".

2 ODE Models

- Give three examples of population models and explain the respective modelling ideas.
- For a given popuation plot, try to guess the underlying population model including the (signs of the) parameters.
- Calculate a point of equilibrium for a given ODE model.
- Explain the different types of equilibrium that can occur. How is the type determined?
- Give an example for models with one (two, no, ...) equilibrium states.
- Describe the "friendly neighbour" situation in the arms-race model
- Characterise possible solutions of a system described by a system of two ODE's (special points, behaviour for $t \to \infty$, draw a rough sketch, ...)
- What is a critical point? What different types are there?
- What are the differences between linear and nonlinear (systems of) ODE(s) with respect to critical points?
- Given a set of plots, and a set of (systems of) ODE(s): match the plots to the ODEs.
- Give some examples on how to include external influences into the ODE models.
- For a given population scenario, suggest one (or several) suitable ODE model(s). If several ODE models are suggested, describe their advantages and disadvantages.
- Give a linear ODE model for a field of crop suffering from enemies. How would the parameters have to be chosen for this system?
- Give/Describe a two-species model that can lead to extinction of one species.
- ● For a given model find the critical points and determine their respective types (using eigenvalues, etc.).
- ∗ Derive a solution of a system of ODEs (with given initial conditions) using the matrix-and-eigenvectors approach.

- ❍ Define the terms homogeneous/non-homogeneous solution of an ODE. Give an example for each.

3 Direction Fields

- What is a direction field – what kind of information does it provide?
- Determine the location of critical points in a direction-field plot.
- Draw a direction field for each of the three types of equilibrium (stable, unstable, saddle point).
- Draw a rough sketch (with critical points and an adequate number of arrows) of the direction field for a given population model.
- Draw a direction field (for a 2D system) that leads to a periodic solution.
- For a given direction field, suggest a model that is compatible with the plot (and give reasons for your suggestion).
- ● Draw a direction fields for a given ODE: $p(t) = f(t, p(t))$. Sketch the different kind solutions that can arise for different initial conditions.

4 Numerical Methods

Numerical schemes for ODE

- Describe the idea to derive of Euler's method to numerically solve ODE's.
- Explain how the Euler schemes (forward and backward) work, for example using a direction field.
- Specify the implicit Euler scheme.
- Given is a (system of) ODE(s): formulate the explicit/implicit Euler's method.
- ● How small do you have to choose the time step such that Euler's scheme is stable for the ODE $\dot{p} = \lambda p$?
- Discuss the advantages and disadvantages of the families of numerical methods for ODE (Runge-Kutta, multistep, implicit)
- What is the difference between the Runge-Kutta method and the Euler method, and why does Runge-Kutta work out better?
- Present the different families of ODE solvers with their stability and performance issues.

- Perform ... steps of a certain numerical scheme to solve a given ODE (by computing values or in a direction field).
- For a given problem: which numerical method is most suitable? Explain why.
- What is the difference between implicit and explicit methods?
- What kind of solver would you suggest for a stiff ODE.
- Describe what problems explicit method have when applied to stiff ODEs.
- Try to find out whether a given model is a discrete model, or whether it results from discretising a continuous model.
- What's the difference between consistency and convergence?

❍ What kind of errors may occur when solvin ODEs numerically?

❍ Describe (a certain) multi-step method for solving ODEs.

Characterisation of ODE schemes

- Explain the terms consistency and convergence of numerical schemes.
- What kind of problems can occur during the numerical solution of ODEs?
- Explain the difference between condition and stability of numerical schemes.
- Explain/Define the terms ill-conditioned and stiffness of ODEs.
- Why are explicit methods unstable for some ODEs?
- What is the *order of convergence*? What does it tell us?
- How can we compute the order of convergence?
- State several factors that affect the efficiency of a numerical scheme.
- What is the problem about too small/too big step sizes in a numerical scheme?