Technische Universität München
Institut für Informatik
Lehrstuhl für numerische Programmierung

Introduction to Scientific Computing

Collection of Exam Questions – ODE

Classification of questions:

- • regular question – could well be part of the exam;
- ● more difficult question;
- ∗ probably too difficult or extensive for the exam;
- ❍ side topic – probably not part of the exam.

The list contains only questions that were generated by the students during the lectures. Consequently, there is no guarantee that the list is complete or representative. However, the questions cover an extensive part of the lecture, and I want to thank everyone who contributed to it.

1 Modelling

- • Define the following terms in the classification of models:
  - discrete vs. continuous
  - determinate vs. stochastic
  - single vs. multivariable (dimensional)
- • Where do the Fibonacci numbers come from?
- ● What is the difference between a discretised model and a discrete model?
- ❍ Explain the “scientific process”.

2 ODE Models

- Give three examples of population models and explain the respective modelling ideas.
- Calculate a point of equilibrium for a given ODE model.
- Explain the different types of equilibrium that can occur. How is the type determined?
- Give an example for models with one (two, no, ...) equilibrium states.
- Describe the "friendly neighbour" situation in the arms-race model
- ❍ Define the terms homogeneous/non-homogeneous solution of an ODE. Give an example for each.

3 Direction Fields

- What is a direction field – what kind of information does it provide?
- Determine the location of critical points.
- Draw a direction field for each of the three types of equilibrium (stable, unstable, saddle point).
- Draw a direction field (for a 2D system) that leads to a periodic solution.
- ● Draw a direction fields for a given ODE: $p(t) = f(t, p(t))$

4 Numerical Methods

Numerical schemes for ODE

- Describe the idea to derive of Euler's method to numerically solve ODE's.
- Explain how the Euler schemes (forward and backward) work.
- Specify the implicit Euler scheme.
- ● How small do you have to choose the time step such that Euler's scheme is stable for the ODE $\dot{p} = \lambda p$?
- Discuss the advantages and disadvantages of the families of numerical methods for ODE (Runge-Kutta, multistep, implicit)

- What is the difference between the Runge-Kutta method and the Euler method, and why does Runge-Kutta work out better?
- Present the different families of ODE solvers with their stability and performance issues.
- Perform ... steps of a certain numerical scheme to solve a given ODE.
- For a given problem: which numerical method is most suitable? Explain why.
- What is the difference between implicit and explicit methods?
- What kind of solver would you suggest for a stiff ODE.

Characterisation of ODE schemes

- Explain the terms consistency and convergence of numerical schemes.
- What kind of problems can occur during the numerical solution of ODEs?
- Explain the difference between condition and stability of numerical schemes.
- Explain/Define the terms ill-conditioned and stiffness of ODEs.
- Why are explicit methods unstable for some ODEs?
- What is the *order of convergence*? What does it tell us?
- How can we compute the order of convergence?
- State several factors that affect the efficiency of a numerical scheme.
- What is the problem about too small/too big step sizes in a numerical scheme?