Scientific Computing I

Module 7: Grid Generation

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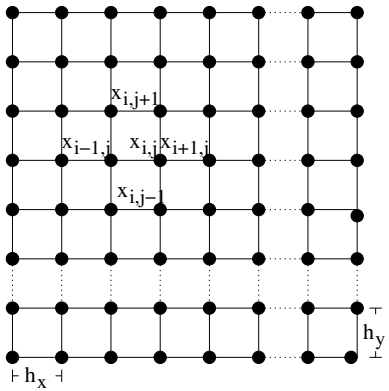
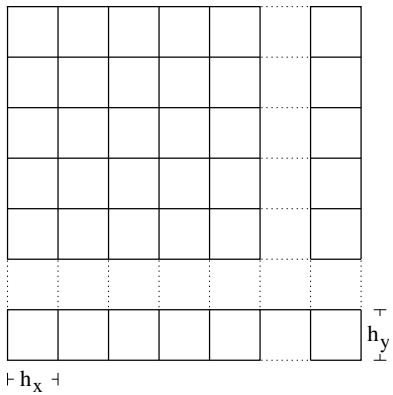
Grid Generation and Refinement

Numerical treatment of PDE

- requires approximate description of the computational domain
- discretization of the domain:
  - *point* discretization (finite differences)
  - *cell* discretization (finite elements/volumes)
- main tasks:
  generating and *refining* grids or meshes;





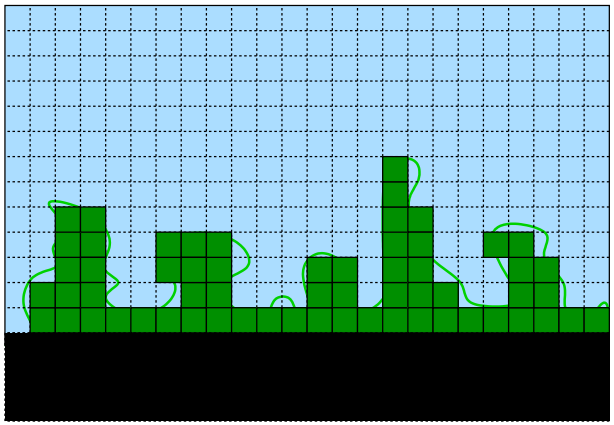
Grid Generation – Objectives

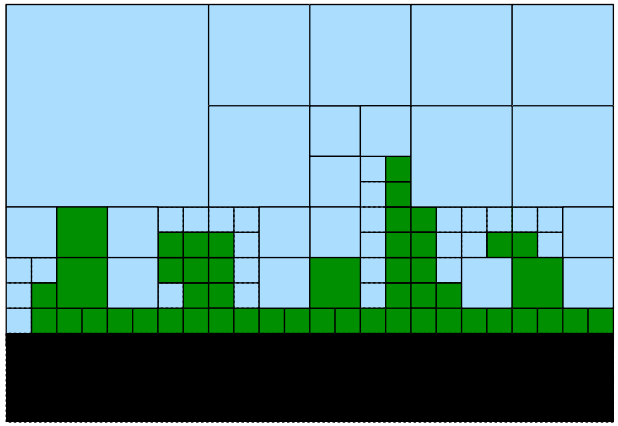
Objectives:

- **accuracy**: accurate (and dense) enough to catch the essential physical phenomena
- **boundary approximation**: sufficiently detailed to represent boundaries and boundary conditions
- **computational efficiency**: small overhead for handling of data structures, no loss of performance on supercomputers
- **numerical adequacy**: features with a negative impact on numerical efficiency should be avoided (angles, distortions)

Structured Grids

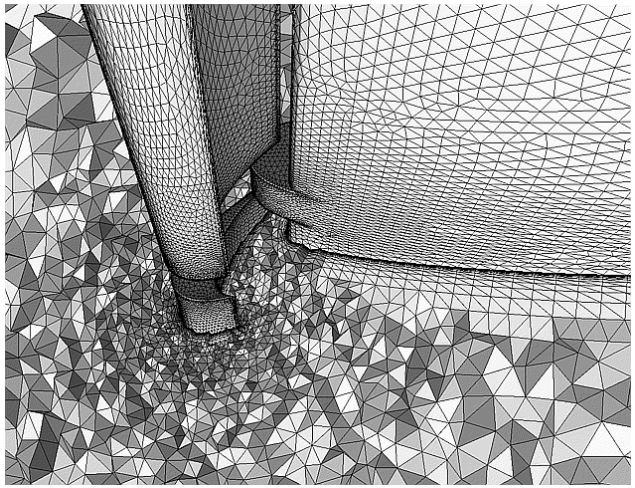
- construction of points or elements follows regular process
- *geometric* (coordinates) and *topological* information (neighbour relations) can be derived





Unstructured Grids

- (almost) no restrictions, maximum flexibilty
- completely irregular generation, even random choice is possible
- explicit storage of basic geometric and topological information



Grid Manipulations

grid generation:

- initial placement of grid points or elements

grid adaption:

- need for (additional) grid points often becomes clear only during the computations
- requires possibilities of both **refinement** and **coarsening**

grid partitioning:

- standard parallelization techniques are based on some *decomposition* of the underlying domain

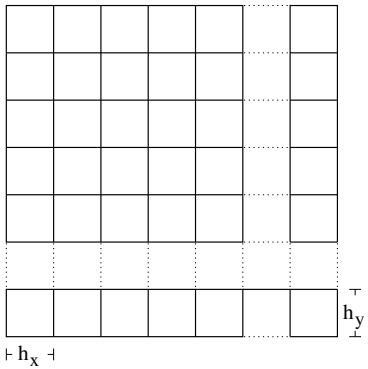
Grid Operations

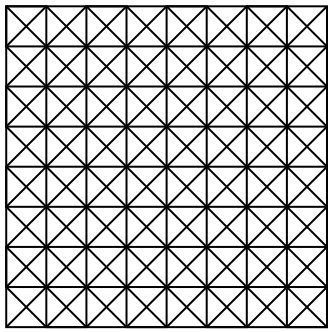
Typical Operations on the Grid:

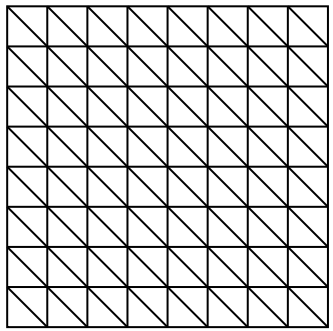
- numbering of the nodes/cells
  - for traversal/processing of the grid
  - for storing the grid
  - for partitioning the grid
- identify the neighbours of a node/cell
  - neighbouring/adjacent nodes/cells/edges
  - due to typical discretization techniques

Regular Structured Grids

- rectangular/*cartesian* grids:
  rectangles (2D) or cuboids (3D)
- triangular meshes:
  triangles (2D) or tetrahedra (3D)
- row-major or column-major traversal and storage

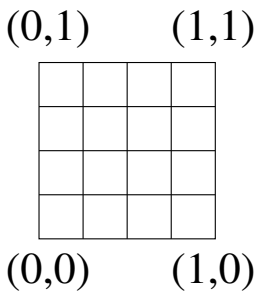


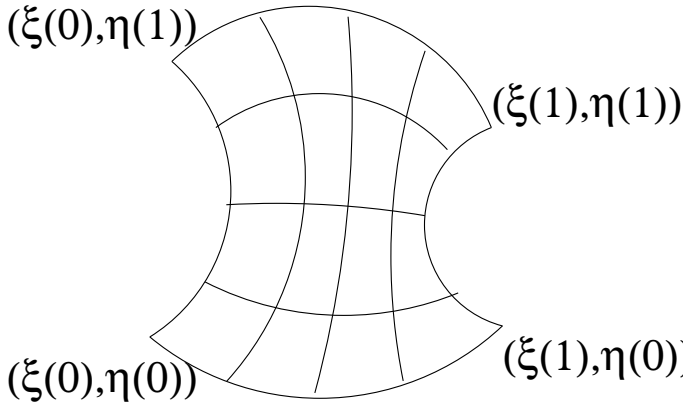




Transformed Structured Grids

- transformation of the unit square to the computational domain
- regular grid is transformed likewise



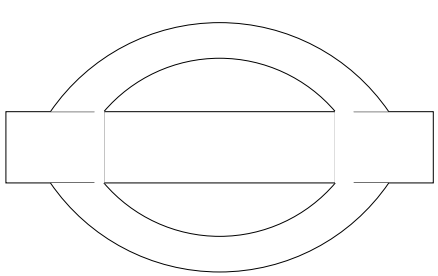


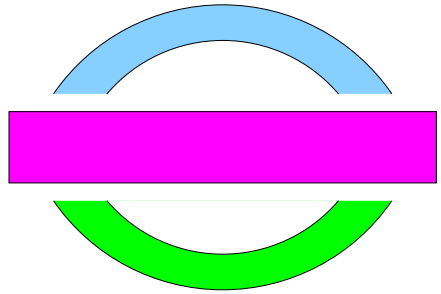
Variants:

- *algebraic*: interpolation-based
  example: *Coons patch*
- *PDE-based*: solve system of PDEs to obtain $(\xi(x,y)$ and $\eta(x,y)$

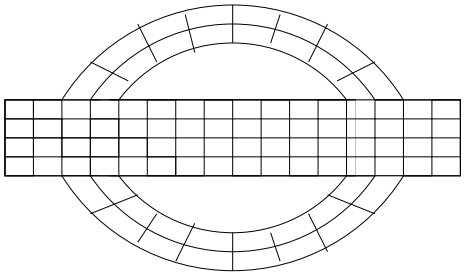
Composite Structured Grids

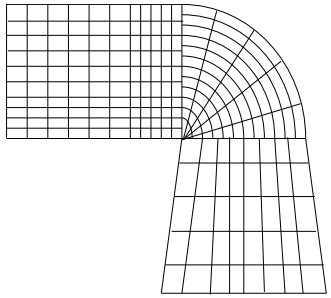
- subdivide (complicated) domain into subdomains of simpler form

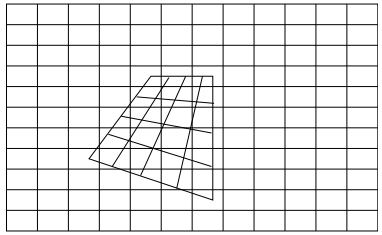




- and use regular meshs on each subdomain

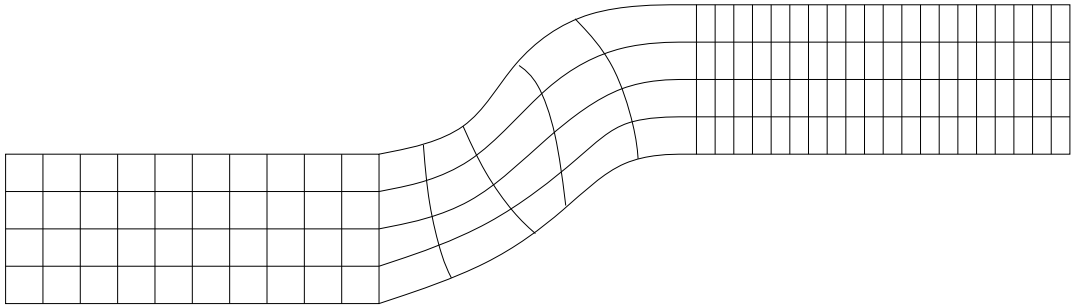






- at interfaces:
  - conforming at interface (“glue” required?)
  - overlapping grids (*chimera* grids)

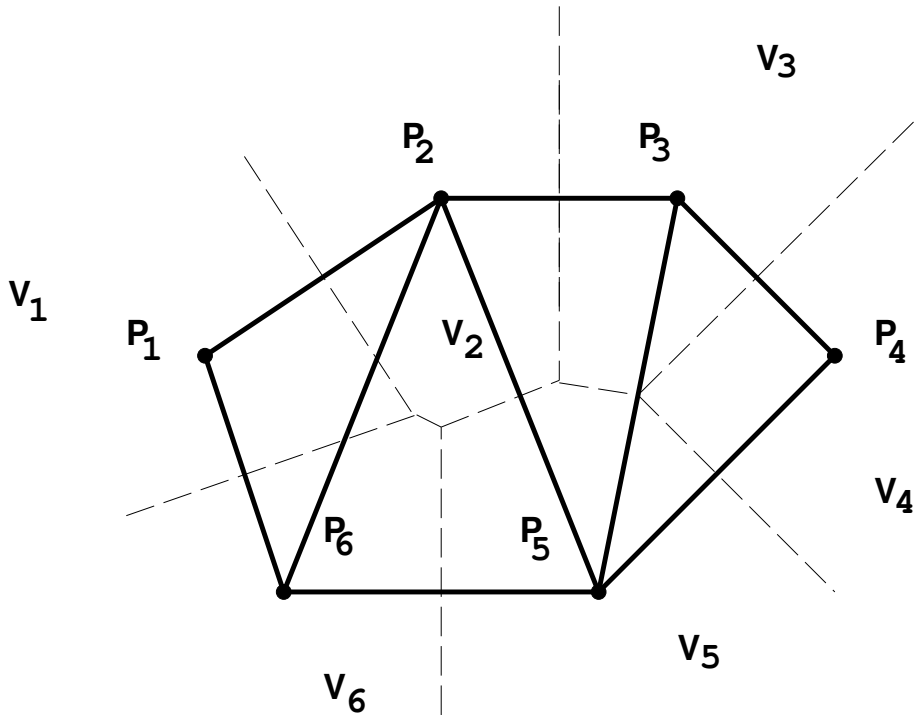
Block Structured Grids



- subdivision into *logically* rectangular subdomains
  (with logically rectangular local grids)
- subdomains fit together in an unstructured way, but continuity is ensured
  (coinciding grid points)
- popular in computational fluid dynamics

Delauney Triangulation

- assume: grid points are already obtained – how to define elements?
- based on **Voronoi** diagrams
- Voronoi region around each given grid point:
  $V_i = \{P : \|P - P_i\| < \|P - P_j\| \ \forall j \neq i\}$



Delauney Triangulation (2)

Algorithm

1. Voronoi region around each given grid point: $V_i = \{P : \|P - P_i\| < \|P - P_j\| \ \forall j \neq i\}$
2. connect points that are located in adjacent Voronoi regions
3. leads to set of disjoint triangles (tetrahedra in 3D)

Applications:

- closely related to FEM, typically triangles/tetrahedra
- very widespread

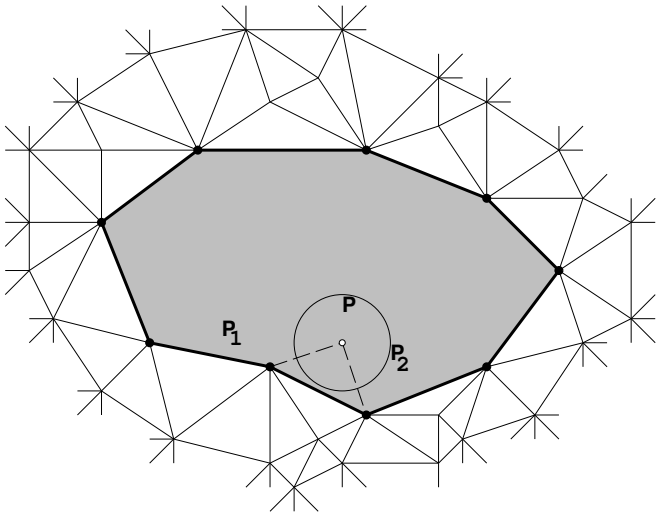
Point Generation

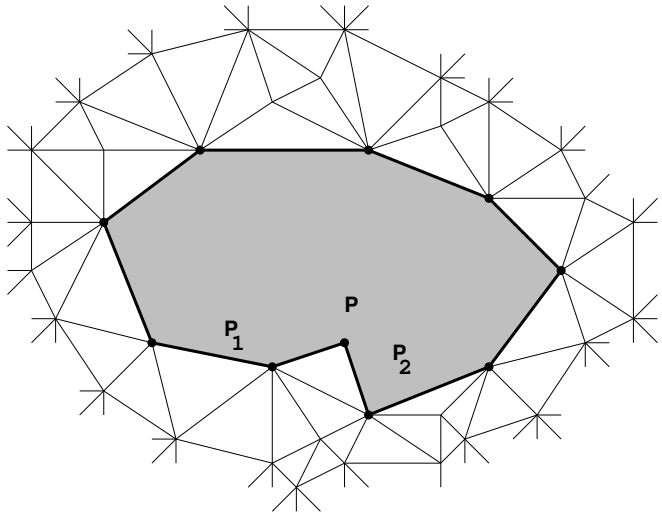
how do we get the grid points?

- start with a regular grid and refine/modify
- boundary-based:
  - start with some boundary point distribution,
  - generate Delaunay triangulation,
  - and subdivide (following suitable rules)
- if helpful, add point or lines sources (singularities, bound. layers)
- Advancing Front methods

Advancing Front Methods

- advance a *front* step-by-step towards interior
- starting from the boundary (*starting front*)





Advancing Front Methods (2)

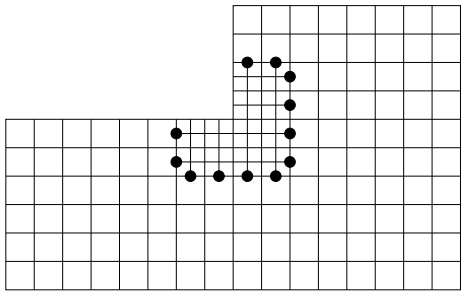
Advancing algorithm:

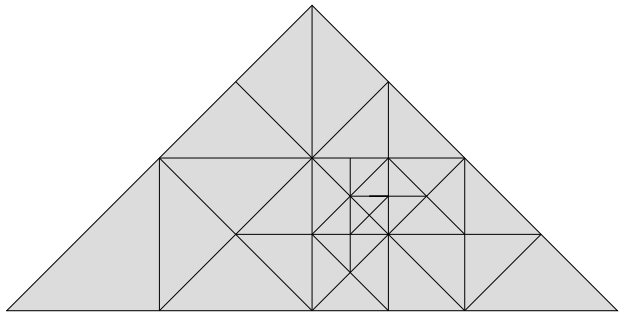
1. choose an edge on the current front, say PQ
2. create a new point R at equal distance d from P and Q
3. determine all grid points lying within a circle around R, radius r
4. order these points w.r.t. distance from R
5. for all points, form triangles with P and Q; select one of these triangles
6. add triangle to grid (unless: intersections, ...)
7. update triangulation and front line: add new cell, update edges

Adaptive Grids

Characterization of adaptive grids:

- size of grid cells varies considerably
- to locally improve accuracy
- sometimes requirement from numerics



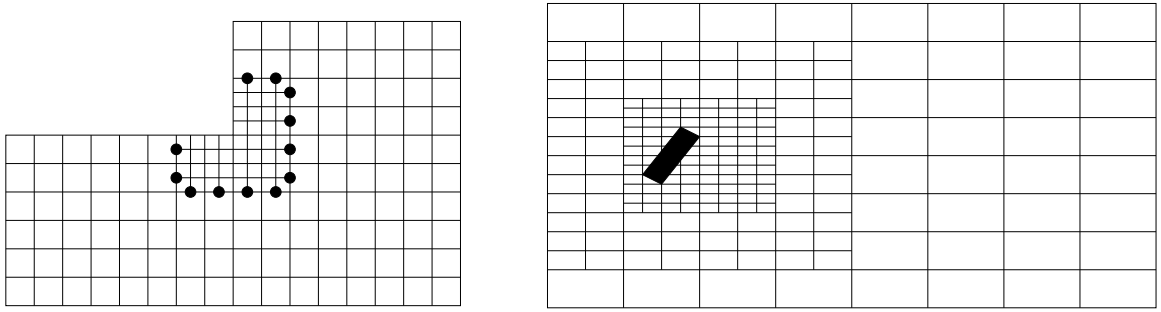


Challenge for **structured** grids:

- efficient storage/traversal
- retrieve structural information (neighbours, etc.)

Block Adaptive Grids

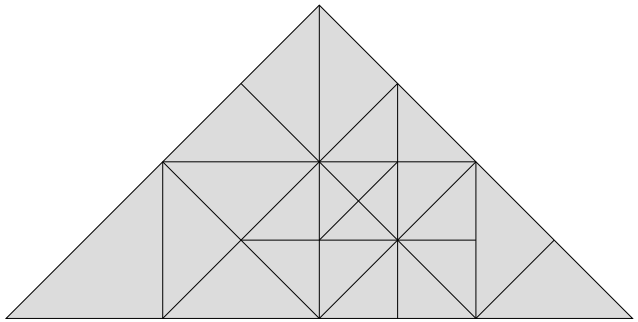
- retain regular structure
- refinement of entire blocks
- similar to block structured grids

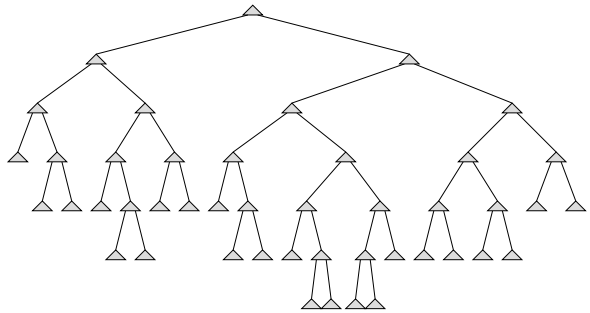


- efficient storage and processing
- but limited w.r.t. adaptivity

Recursively Structured Adaptive Grids

- based on recursive subdivision of parent cell(s)
- leads to tree structures
- quadtree/octree or substructuring of triangles:





- efficient storage; flexible adaptivity
- but complicated processing (recursive algorithms)