Technische Universität München
Institut für Informatik
Dr. Miriam Mehl
Dr. Michael Bader

WT 2004/2005

December 22, 2004

Introduction to Scientific Computing

Final Exam, February 9th 2005

1 Population Dynamics

Consider the population model

$$\dot{p} = f(p) = -r \cdot \left(1 - \frac{p}{k}\right) \cdot \left(1 - \frac{p}{l}\right)$$

with $p(0) = p_0 \geq 0$ and $r > 0, \quad 0 < l < k$.

a) Compute the critical points! Are they stable or not?

b) Determine the limit $\lim_{t \to \infty} p(t)$ for

I) $0 \leq p_0 < l$,
II) $p_0 = l$,
III) $l < p_0 < k$,
IV) $p_0 = k$.
V) $p_0 > k$.

c) Compute the population p_s with

$$\frac{df}{dp}(p_s) = 0.$$

d) Sketch a trajectory of the solution $p(t)$ for each of the five cases from **b)**.

e) Name one reason why the model is not a realistic population model.

2 Explicit Midpoint Rule

Consider the explicit midpoint rule

$$y_{k+1} = y_k + \delta t \cdot f\left(t_k + \frac{\delta t}{2}, y_k + \frac{\delta t}{2} f(t_k, y_k)\right)$$

used for the approximate solution of the initial value problem $\dot{y} = f(t, y), \quad y(0) = y_0$.

a) Show that the midpoint rule is at least second order consistent.

b) Is the midpoint rule convergent? Give a lower limit for the order of convergence.

c) Under which restrictions for δt is the stability condition

$$|y_k| \leq |y_0| \text{ for all } k$$

fulfilled for $f(t, y) = \lambda \cdot y$ with $\lambda < 0$?