Introduction to Scientific Computing II

Tutorial April 23, 2012

Tutorial

Wolfgang Eckhardt
Email: eckhardw@in.tum.de
Room: 02.05.55

Time of tutorial: Monday, 9:00 - 9:45 am

URL of Lecture / Tutorial: `http://www5.in.tum.de/wiki/index.php/Introduction_to_Scientific_Computing_II_-_Summer_12`

Overview

Matlab

Finite Difference Discretization of the Poisson-Equation

1.1. Matlab

- All exercises in Matlab
- gain understanding of mathematical methods / numerical algorithms (i.e. no Matlab programming course)

- Rechnerhalle / lxhalle
  - login to lxhalle.informatik.tu-muenchen.de
  - $ /mount/applic/packages/matlab/bin/matlab
- for free: Floating Licenses for TUM students (see Webpage)

- Octave / QtOctave also ok
- available in most linux distributions prepackaged

Matlab - Getting help

- Online resources:
  - Online help:
    `www.mathworks.com/access/helpdesk/help/techdoc/matlab.html`
  - Getting Started Guide and interactive tutorials: `www.mathworks.com/academia/student_center/tutorials/launchpad.html`
  - Matlab Primer:
    `http://math.ucsd.edu/~driver/21d-s99/matlab-primer.html`
- Matlab built-in help: type **help** or **help KEYWORD**

1.2. Finite Difference Discretization of the Poisson-Equation

We want to solve the **Poisson-Equation**:

$$\Delta u(x) = f(x), \quad x \in \Omega, \quad \Omega \subseteq \mathbb{R}^n$$

with **Dirichlet-Boundary-Conditions**

$$u(x) = 0, \quad x \in \partial\Omega$$

The **Laplacian operator** is defined as

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \ldots + \frac{\partial^2}{\partial x_n^2}$$

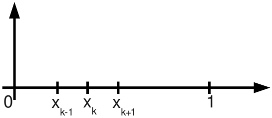
Solve 1-d Poisson-Equation on unit intervall:

$$\frac{\partial^2}{\partial x^2} u(x) = 0, \quad x \in]0; 1[\,, \quad u(0) = 0,\ u(1) = 0;$$

Usually, solutions of PDEs can't be determined analytically

$\Rightarrow$ calculate numerically for certain values.

Finite Difference Discretization



Deduced from Taylor-Series in point x

$$(I)\ u(x_k + h) = u_{k+1} = u(x_k) + hu'(x_k) + \frac{h^2}{2}u''(x_k) + O(h^3)$$

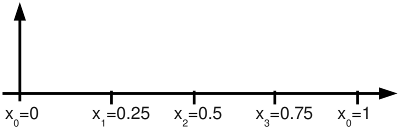
$$(II)\ u(x_k - h) = u_{k-1} = u(x_k) - hu'(x_k) + \frac{h^2}{2}u''(x_k) - O(h^3)$$

$(I) - (II)$, solve by $u'(x_k)$:

$$u'(x_k) = \frac{u(x_{k+1}) - u(x_{k-1})}{2h}$$

$(I) + (II)$, solve by $u''(x_k)$:

$$u''(x_k) = \frac{u(x_{k+1}) - 2u(x_k) + u(x_{k-1})}{h^2}$$



Solve 1-d Poisson-Equation on unit intervall:

$$\frac{\partial^2}{\partial x^2} u(x) = 0, \quad x \in]0;1[\,, \quad u(0) = 0,\ u(1) = 0;$$

Approximation: $u''(x_k) = \frac{u(x_{k-1}) - 2u(x_k) + u(x_{k+1})}{h^2}$

k=1: $u''(x_1) = \frac{u(x_0) - 2u(x_1) + u(x_2)}{h^2} = b_1$

k=2: $u''(x_2) = \frac{u(x_1) - 2u(x_2) + u(x_3)}{h^2} = b_2$

k=3: $u''(x_3) = \frac{u(x_2) - 2u(x_3) + u(x_4)}{h^2} = b_3$