Technische Universität München
Institut für Informatik
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ST 2012

Scientific Computing II

Homework Exercise 1

April 30, 2012 - due by May 4, 8 am

"Hausaufgaben-Wichteln": Doing a "Secret Santa"

- Do (at least ;)) the task you will be assigned
- Hand in your solution (both plot and matlab code) per email to `eckhardw@in.tum.de` by Friday morning, clearly stating which exercise you solved.
- On Monday, the solutions will be discussed. Everyone, who handed in a solution, will be given the solution of someone else to correct and / or to discuss it.
- The best solutions handed in for each task will be compiled into one document that will be made available at the website of the lecture.
- In case you were not assigned a task in the tutorial, you can participate nevertheless: each of the tasks **c, d, e, f** has to be done for the solvers **1 - 6**, so you could decide to do e.g. task **d** for solver **4**.

Iterative Solvers

We have to solve the discretised two-dimensional Poisson equation on the unit interval with homogeneous Dirichlet boundary conditions:

$$\begin{aligned} \Delta u &= 0 \text{ in }]0,1[^2, \\ u &= 0 \text{ at } \partial]0,1[^2. \end{aligned}$$

The Laplacian is discretized in a regular grid by the 5-point-stencil

$$\Delta u(i \cdot h, j \cdot h) \approx \frac{u_{i-1,j} + u_{i,j-1} - 4u_{i,j} + u_{i+1,j} + u_{i,j+1}}{h^2}.$$

We solve the resulting system of linear equations using

1) Jacobi

2) Damped Jacobi with $\omega = \frac{1}{2}$

3) Damped Jacobi with $\omega = \frac{2}{3}$

4) Gauss-Seidel

5) Red-Black Gauss-Seidel

6) SOR with the optimal overrelaxation factor $\omega = \frac{2}{1+\sin(\pi h)}$

Let n denote the number of inner grid points per dimension (i.e. the total number of degrees of freedom is n^2, and the meshwidth $h = \frac{1}{n+1}$). As initial guess, we use

$$u^{initial}(i,j) = sin(\pi hi) * sin(\pi hj) + sin(\frac{n}{2}\pi hi) * sin(\frac{n}{2}\pi hj) + sin(n\pi hi) * sin(n\pi hj).$$

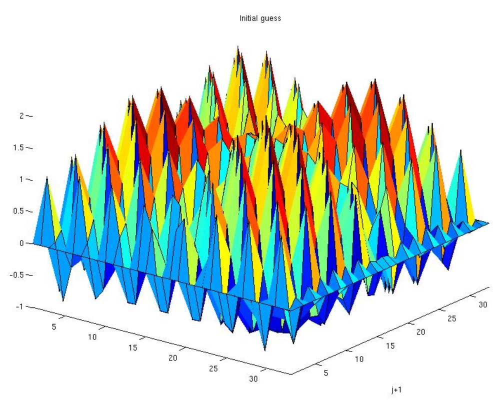


Figure 1: Initial guess on a grid with $n = 31$.

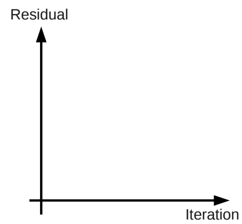
Remarks:

- The exact solution for the given system is $u = 0$ such that our current approximation at the same time represents the current error.
- Use the code skeleton available at the web page of the lecture.

a) Establish the formula for the update of the value u_i at grid point i for all solvers.

b) Implement a Matlab programme solving the PDE with the given methods.

c) Create 2 plots of the current solution after 10 iterations on a grid with $n = 15$ and $n = 127$ for the given solvers.



d) Perform iterations until the residual error is less than 10 on a grid with $n = 15$ and $n = 127$. Create a plot for each solver showing the residual error in the l2- norm in each iteration. (**Hint:** Use the given function `max_residual()`.)

e) Perform iterations until the residual error in the l2-norm is less than 1.0. Create a plot for each solver for the number of iterations needed on a grid with $n = 7, 15, 31, 63, 127, 255$. How does the number of iterations behave? Use the O(N)–notation.

f) Now we want to study, how errors corresponding to different eigenmodes are reduced. As initial guess, we use

$$u_k^{initial}(i,j) = sin(k\pi hi) * sin(k\pi hj)$$

For each solver, perform 1 iteration on a grid with $n = 31$ for all possible $k \in [1, ..., n]$ as initial guess. Create a plot of the error reduction factor for the different wave numbers.