Technische Universität München
Institut für Informatik
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Wolfgang Eckhardt

ST 2012

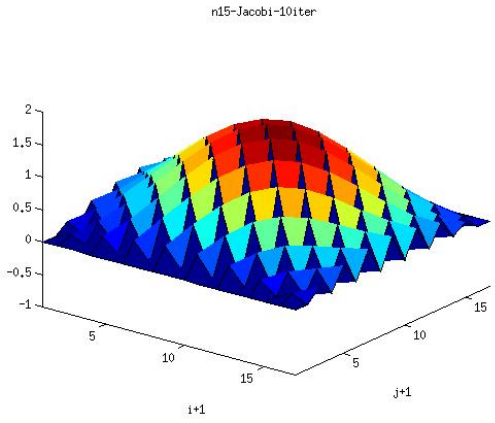
Scientific Computing II

Homework Exercise 1

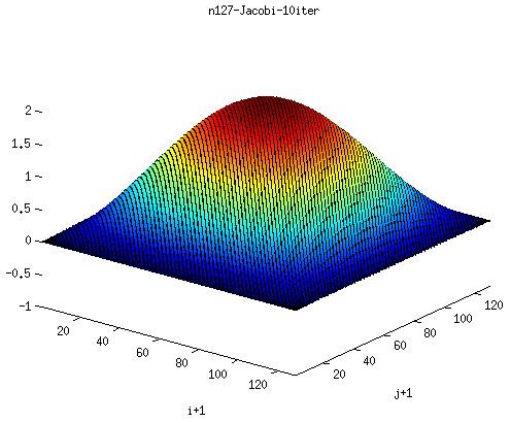
April 30, 2012 - due by May 4, 8 am

Iterative Solvers – Solutions

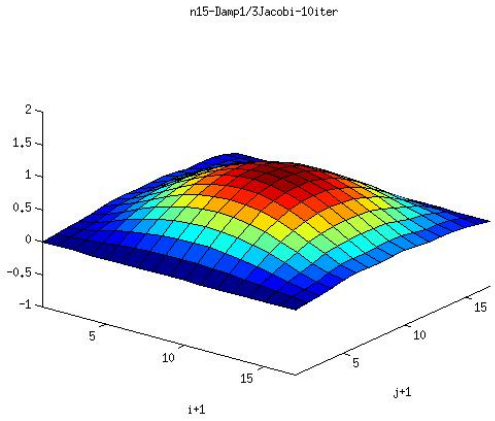
c) Create 2 plots of the current solution after 10 iterations on a grid with $n = 15$ and $n = 127$ for the given solvers.



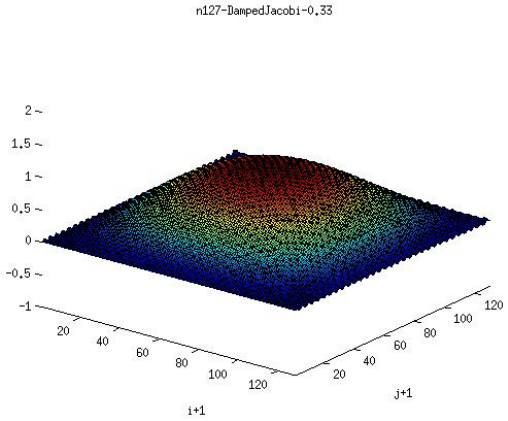
(a) Jacobi - $n = 15$.



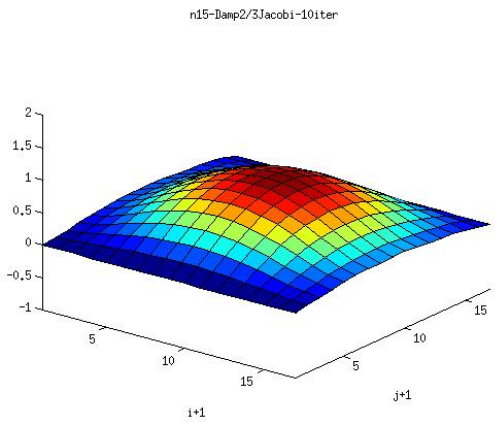
(b) Jacobi - $n = 127$.



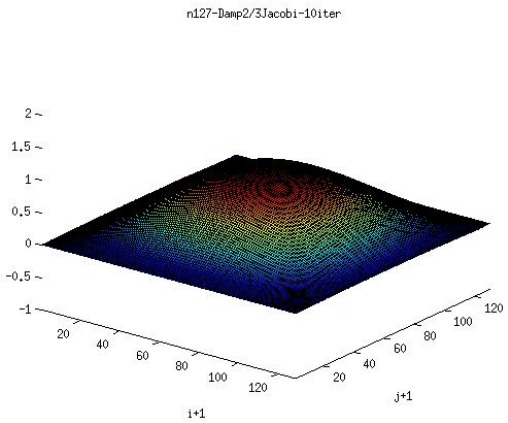
(c) Damped Jacobi $\omega = 0.33$ - $n = 15$.



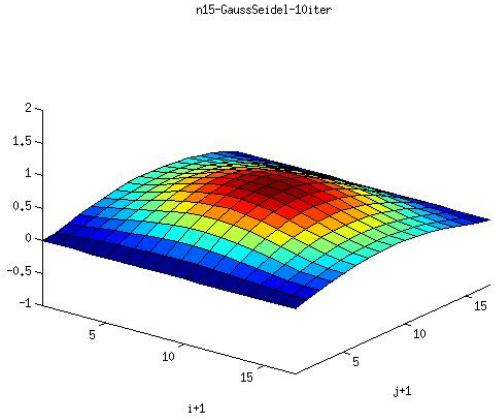
(d) Jacobi $\omega = 0.33$ - $n = 127$.



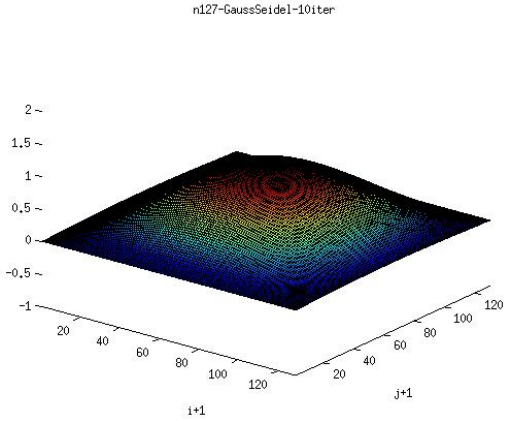
(e) Damped Jacobi $\omega = 0.66$ - $n = 15$.



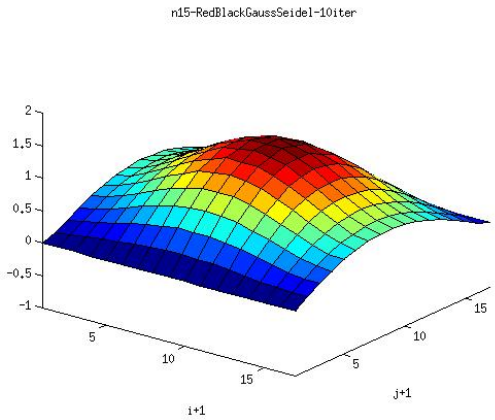
(f) Jacobi $\omega = 0.66$ - $n = 127$.



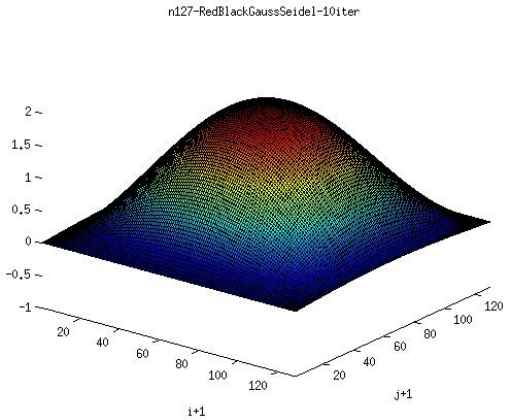
(g) Gauss–Seidel - $n = 15$.



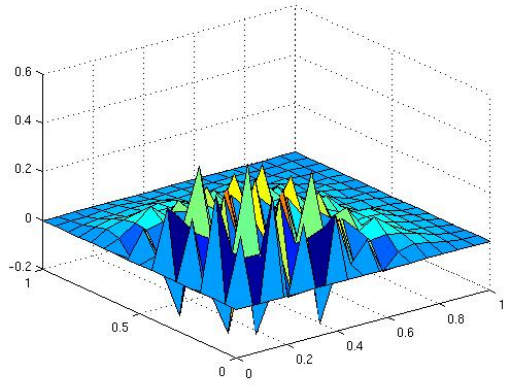
(h) Gauss–Seidel -$n = 127$.



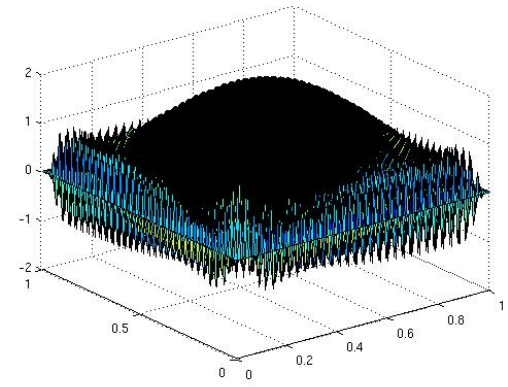
(i) Red–Black Gauss–Seidel - $n = 15$.



(j) Red–Black Gauss–Seidel - $n = 127$.

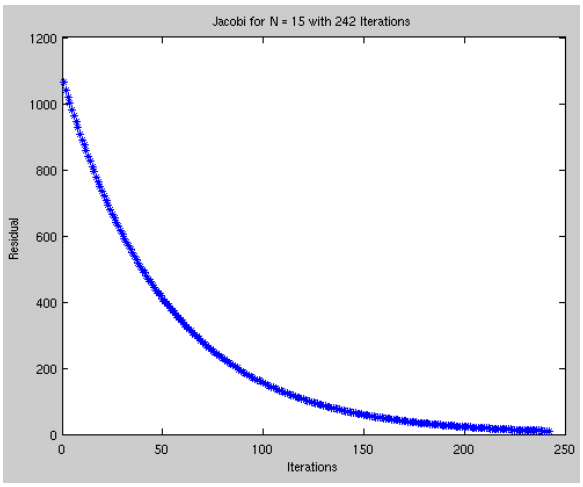


(k) SOR - $n = 15$.

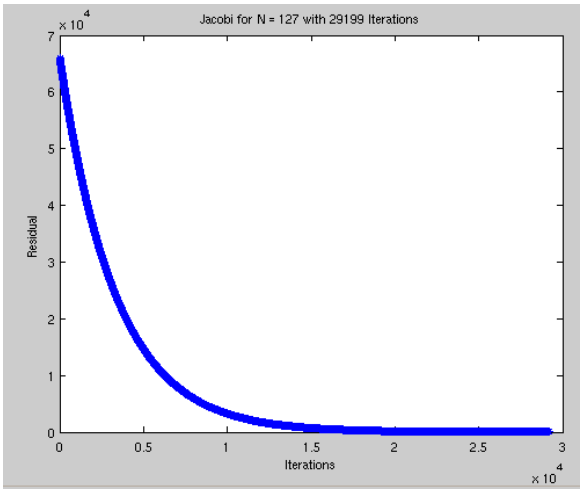


(l) SOR - $n = 127$.

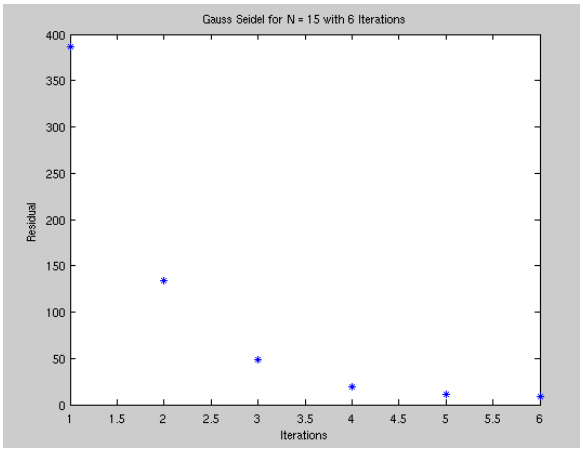
d) Perform iterations until the residual error is less than 10 on a grid with $n = 15$ and $n = 127$. Create a plot for each solver showing the residual error in the l2- norm in each iteration. (**Hint:** Use the given function `max_residual()`.)



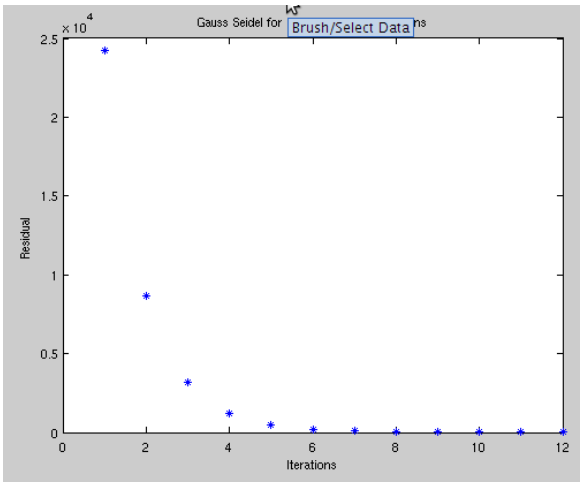
(m) Jacobi - $n = 15$.



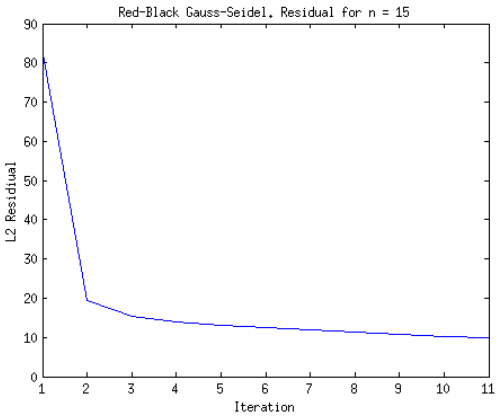
(n) Jacobi - $n = 127$.



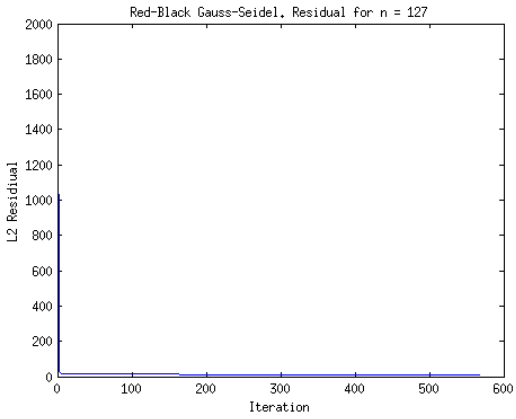
(o) Gauss–Seidel - $n = 15$.



(p) Gauss–Seidel - $n = 127$.



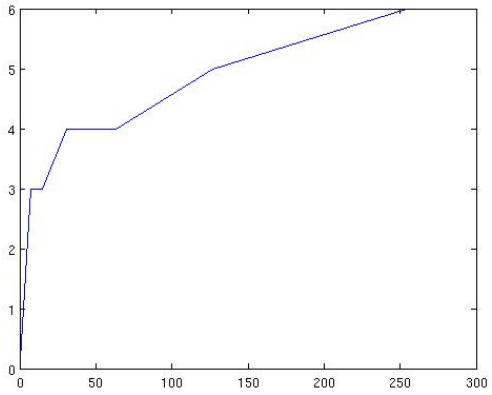
(q) Red–Black Gauss–Seidel - $n = 15$.



(r) Red–Black Gauss–Seidel - $n = 127$.

e) Perform iterations until the residual error in the l2-norm is less than 1.0. Create a plot for each solver for the number of iterations needed on a grid with $n = 7, 15, 31, 63, 127, 255$. How does the number of iterations behave? Use the O(N)–notation.

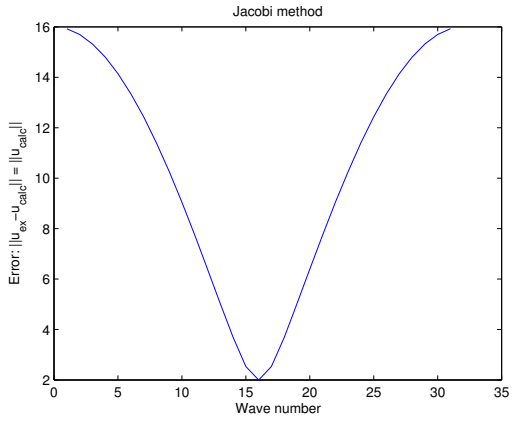
Red-Black Gauss-Seidel:



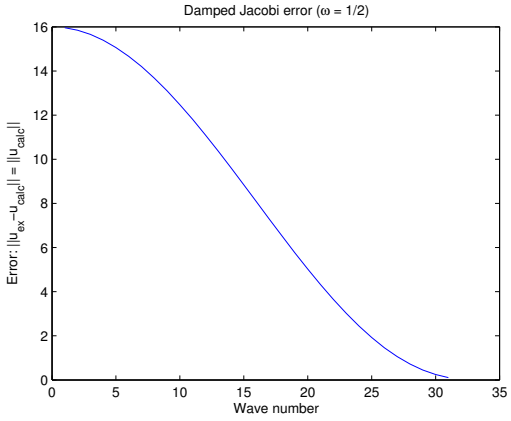
f) Now we want to study, how errors corresponding to different eigenmodes are reduced. As initial guess, we use

$$u_k^{initial}(i,j) = sin(k\pi hi) * sin(k\pi hj)$$

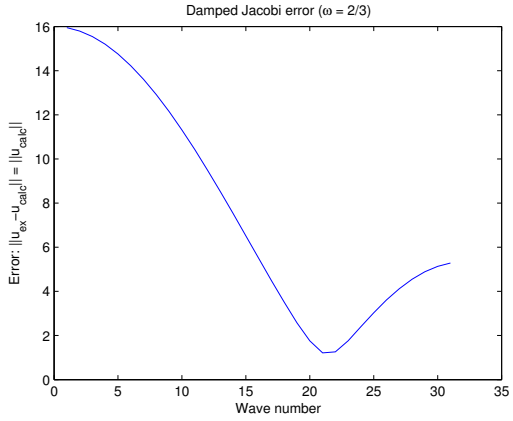
For each solver, perform 1 iteration on a grid with $n = 31$ for all possible $k \in [1, ..., n]$ as initial guess. Create a plot of the error reduction factor for the different wave numbers.



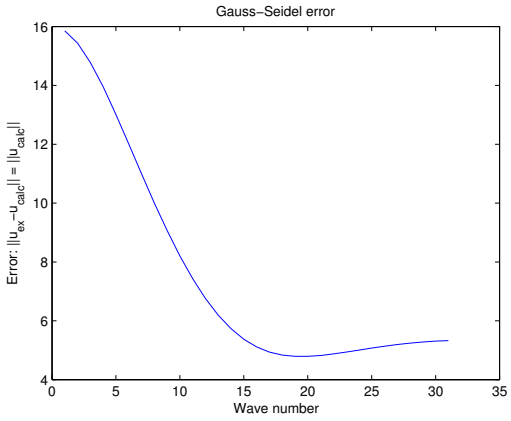
(s) Jacobi.



(t) Damped Jacobi $\omega = 0.5$.



(u) Damped Jacobi $\omega = 0.66$.



(v) Gauss-Seidel.