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# Scientific Computing II

## Exercise 2

**May 14, 2012**

## Homework: Multigrid

We have to solve the discretised three-dimensional Poisson equation on a unit cube with homogeneous Dirichlet boundary conditions:

$$\begin{aligned} \Delta u &= f \text{ in } ]0;1[^3, \\ u &= 0 \text{ at } \partial]0;1[^3. \end{aligned}$$

The Laplacian is discretised in a regular cartesian grid by the 7-point-stencil

$$\Delta u(i \cdot h, j \cdot h, k \cdot h) \approx \frac{u_{i-1,j,k} + u_{i,j-1,k} + u_{i,j,k-1} - 6u_{i,j,k} + u_{i+1,j,k} + u_{i,j+1,k} + u_{i,j,k+1}}{h^2}.$$

**a)** The given `main`-program executes a two-grid solver for the system described above. It uses Gauss-Seidel as a smoother with two pre- and two postsmoothing iterations. Record the resulting runtimes and numbers of iterations in the tabular in **b)**.

Write the number of iterations in the form $O(N^p)$ with a suitable $p$. Is the qualitative behaviour of the number of iterations optimal?

How can you explain the strong increase of the runtime?

**b)** Modify the two-grid solver such that you obtain a multigrid v-cycle. Solve our system with this modified program and record the resulting runtimes and numbers of iterations in the second of the following tabulars:

**Two-grid method**

| $N$ | runtime | # iterations |
|---|---|---|
| 7 | sec | |
| 15 | sec | |
| 31 | sec | |

**Multigrid method (V-cycle)**

| $N$ | runtime | # iterations |
|---|---|---|
| 7 | sec | |
| 15 | sec | |
| 31 | sec | |
| 63 | sec | |
| 127 | sec | |

Do you observe any significant differences in comparison to the two-grid solver (in particular concerning the resulting numbers of iterations)?

**c)** The overall costs of the multigrid solver are $O\left(N^{q_{MG}}\right)$ Determine $q_{MG}$ and compare the results to the costs of Gaussian elimination, Jacobi, Gauss-Seidel, and the SOR method.