Technische Universität München
Institut für Informatik
Prof. Dr. Michael Bader
Wolfgang Eckhardt

ST 2012

Scientific Computing II

Exercise 4

June 11, 2012

Tutorial: Conjugate Gradients Method

We have to solve the discretised three-dimensional Poisson equation on a unit square with homgeneous Dirichlet boundary conditions (see exercise 1) and constant right-hand side one. The Laplacian is discretised by the known 7-point-stencil (see exercise 1).

a) Give the costs per iteration of the conjugate gradient method!

b) Give the overall costs of the conjugate gradient method if used as a direct solver! Compare to the results for Gaussian elimination!

c) Use the asymptotic convergence rate of the conjugate gradient method (computed in the lecture) to guess the costs of the method if used as an iterative solver!

Hint: Give the costs in the form $O(N^q)$ if N is the number of unknowns per coordinate direction.

Homework: Steepest Descent

We have to solve the discretised three-dimensional Poisson equation on a unit square with homgeneous Dirichlet boundary conditions (see exercise 1). The Laplacian is discretised by the known 7-point-stencil (see exercise 1).

a) Give the costs per iteration of the steepest decent in the form $O(N^p)$!

b) Implement a matlab function performing one steepest descent iteration for the given system in dependence on the grid resolution per coordinate direction N, the right hand side b, and the current solution approximation u.

Hint: Use the given function `residual_vec` to compute the current residual vector.

c) If you solve the given system (using the given `main`-function) with the steepest descent method for different grid resolutions N you will notice that you are always done with one iteration. Thus, for this example, the first steepest descent direction directly leads to the solution. To see that this doesn't hold in general, change the right-hand side to one (constant function) and try to solve the system again. Record the resulting runtimes and numbers of iterations in the following tabular:

N	runtime	# iterations
7	sec	
15	sec	
31	sec	

d) Can you give a relation between N and the number of iterations required with the steepest descent in the form $O(N^p)$?

e) Give the overall costs of the steepest descent method for the given system in the form $O(N^q)$. Do you know solvers that have the same complexity?