4. Krylov Methods

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Reference

J.R. Shewchuk:

An Introduction to the Conjugate Gradient Method Without the Agonizing Pain

Describes **Steepest Descent** and **CG** (at least with not so much pain).

Steepest Descent - Algorithm

Steepest Descent - Algorithm

- Choose

$$\vec{r} = \vec{b} - A\vec{u}$$

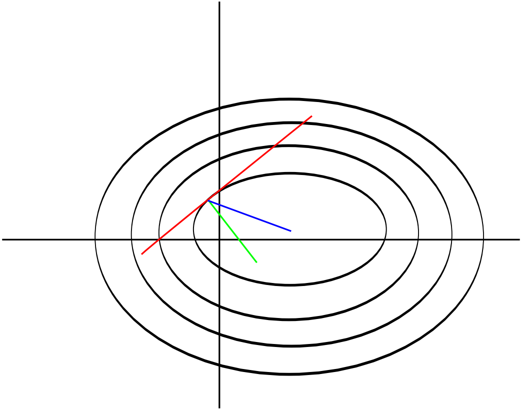
  as search direction if the current approximation is $\vec{u}$,
- Choose

$$\alpha = \frac{\vec{r}^T\vec{r}}{(A\vec{r})^T\vec{r}}$$

  as stepsize,
- New approximation:

$$\vec{u}^{new} = \vec{u} + \alpha\vec{r}.$$

Conjugate Gradients - Idea



We could make the individual search directions be orthogonal to each other. Then, we end up with a direct solver. However, we should not use the standard orthogonality (green), but vector product induced by A (blue).

Conjugate Gradients - Algorithm

How far to go along d_i?	α	=	$\frac{r_i^T d_i}{d_i^T A d_i}$
Compute new u_i	u_{i+1}	=	$u_i + \alpha d_i$
Compute new residual	r_{i+1}	=	$r_i - \alpha A d_i$
Compute β, so that	β	=	$\frac{r_{i+1}^T r_{i+1}}{r_i^T r_i}$
d_i and d_{i+1} are orthogonal	d_{i+1}	=	$r_{i+1} - \beta d_i$