

Scientific Computing II

Exercise 5

June 18, 2012

Homework: Preconditioned Conjugate Gradients Method

We have to solve the discretised three-dimensional Poisson equation on a unit square with homogeneous Dirichlet boundary conditions (see exercise 1) and constant right-hand side one. The Laplacian is discretised by the known 7-point-stencil.

- a) Implement a matlab function performing the preconditioning in the context of the preconditioned conjugate gradient method for the given system in dependence on the grid resolution per coordinate direction N , the current residual and the type of preconditioner (1 =Jacobi or 2 =Gauss-Seidel as shown in the lecture).
- b) Implement a matlab function performing one iteration of the preconditioned conjugate gradient method for the given system in dependence on the grid resolution per coordinate direction N and the current solution approximation u .
- c) Solve the given system with the help of the given `main`-program using the `pcg` method for different grid resolutions N and with both preconditioners. Record the resulting runtimes and numbers of iterations in the following tabular:

N	runtime (Jacobi)	runtime (Gauss-Seidel)	# it (Jacobi)	# it (Gauss-Seidel)
7	sec	sec		
15	sec	sec		
31	sec	sec		
63	sec	sec		
127	sec	sec		

- d) Compare your results to the conjugate gradient method without preconditioning and the optimal convergence of a multigrid method.