

# Scientific Computing II

## Exercise C

July 9, 2012

### Tutorial: Time Discretisation

In the following, we consider the differential equation

$$\frac{d^2}{dt^2}\vec{r}(t) = -\vec{r}(t).$$

- a) Discuss possible solutions of this differential equation. How many initial conditions need to be given to make the solution unique? Do the solutions differ for the 1D-, 2D- or 3D-case?
- b) Transform the 1D-equations in a system of 1st order differential equations and apply the numerical methods known to you: explicit and implicit Euler, trapezoidal rule (Crank-Nicholson), mid-point rule, Störmer-Verlet and its Leap-frog variant.
- c) For the single step methods, formulate one timestep as matrix-vector product. What can you conclude for the longterm behaviour of the methods? Calculate the eigen vectors or their product respectively for each method.

**Hint: The product of the eigenvalues can also be calculated from the determinant.**

(Cf.: G. Strang, “Computational Science and Engineering”, Kap. 2.2)