



Introduction to Scientific Computing II

From Relaxation to Multigrid

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Multigrid – Principles

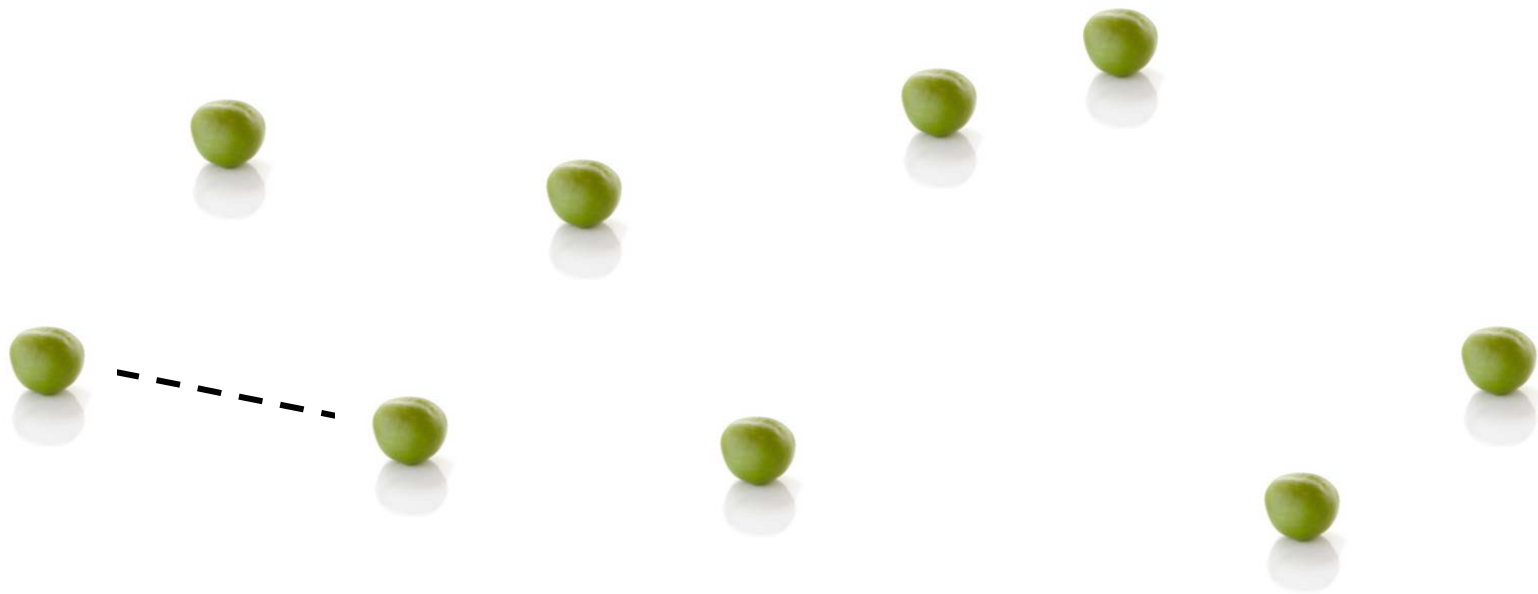
- fine grid
 - eliminate high frequencies
- coarse grids
 - eliminate low frequencies(!)
 - equation for the error(!)
 - error smooth => representable

Multigrid – Algorithm

- iterate (GS) on the fine grid
- restrict residual to the coarse grid
- solve coarse grid equation for the error
- interpolate error to the fine grid
- correct fine grid solution

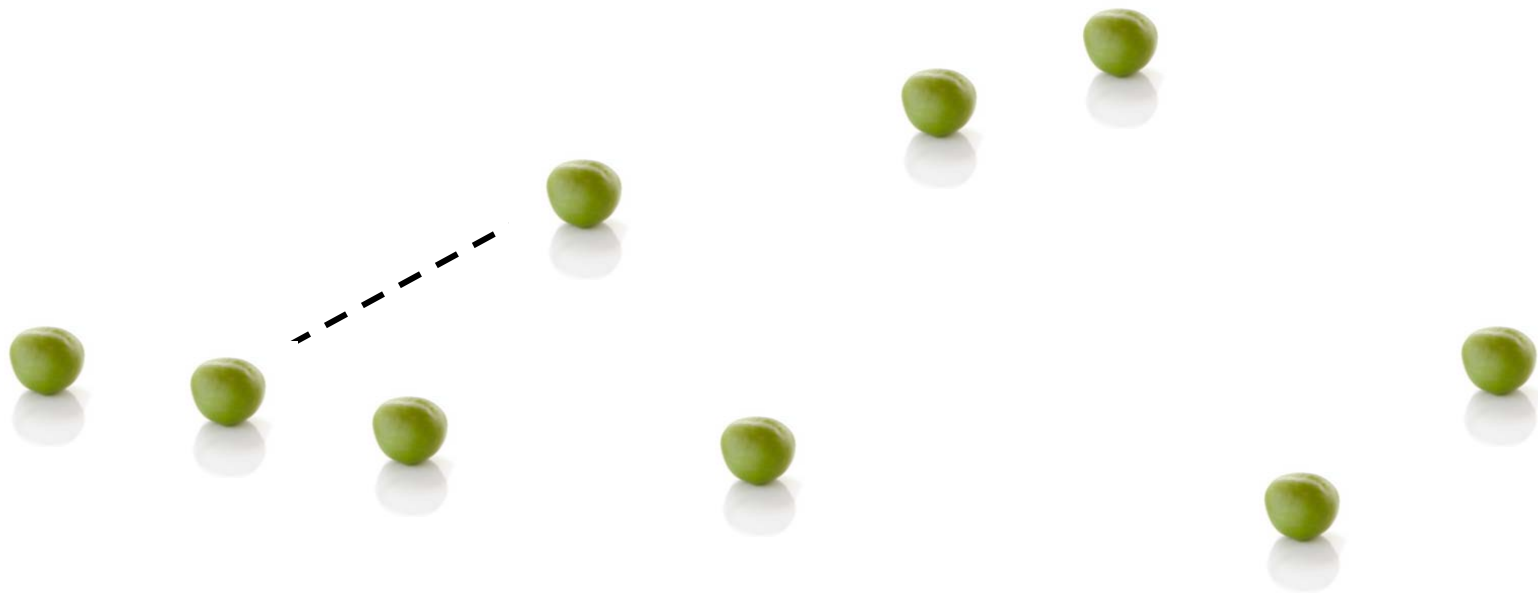
Multigrid Methods – Presmoothing

Gauss Seidel



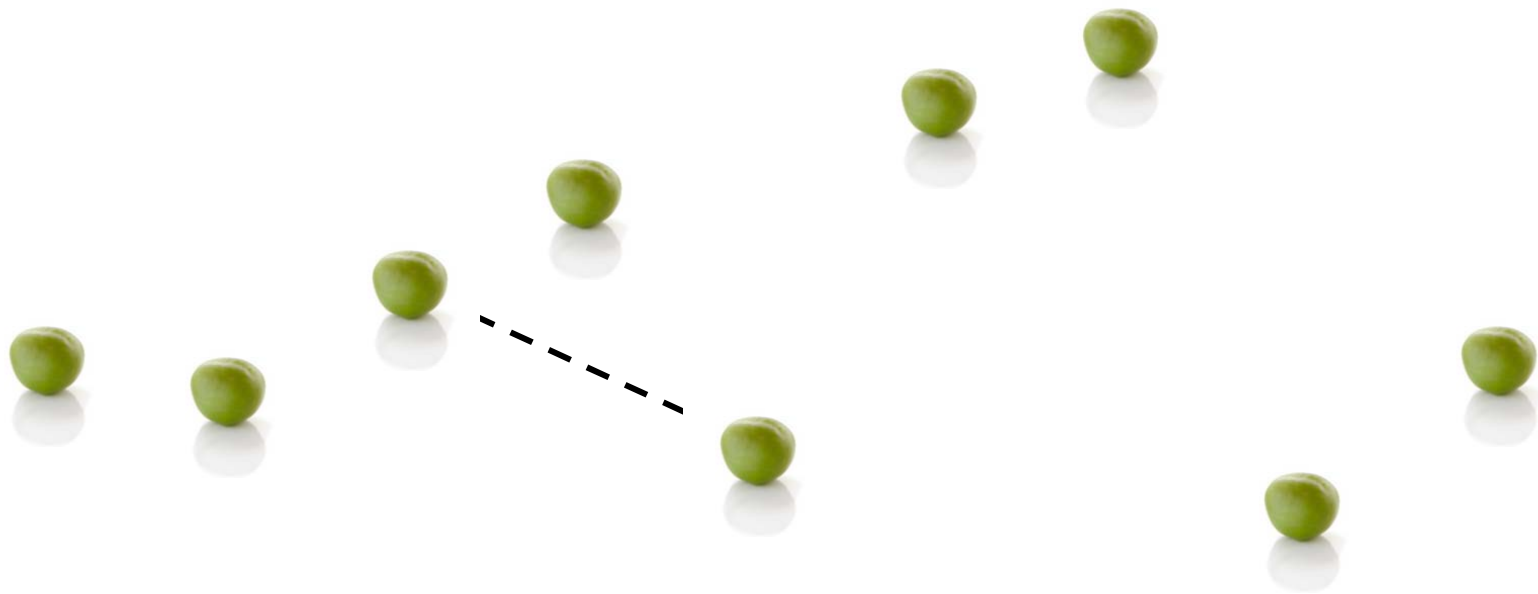
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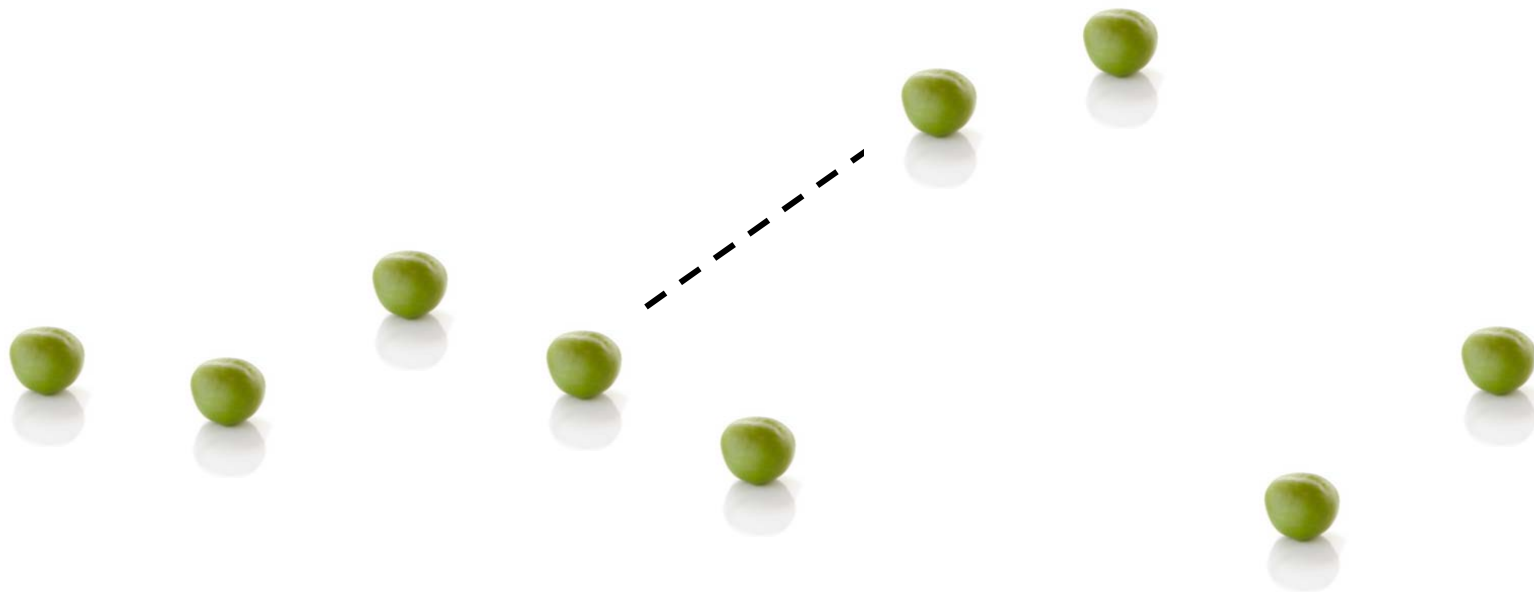
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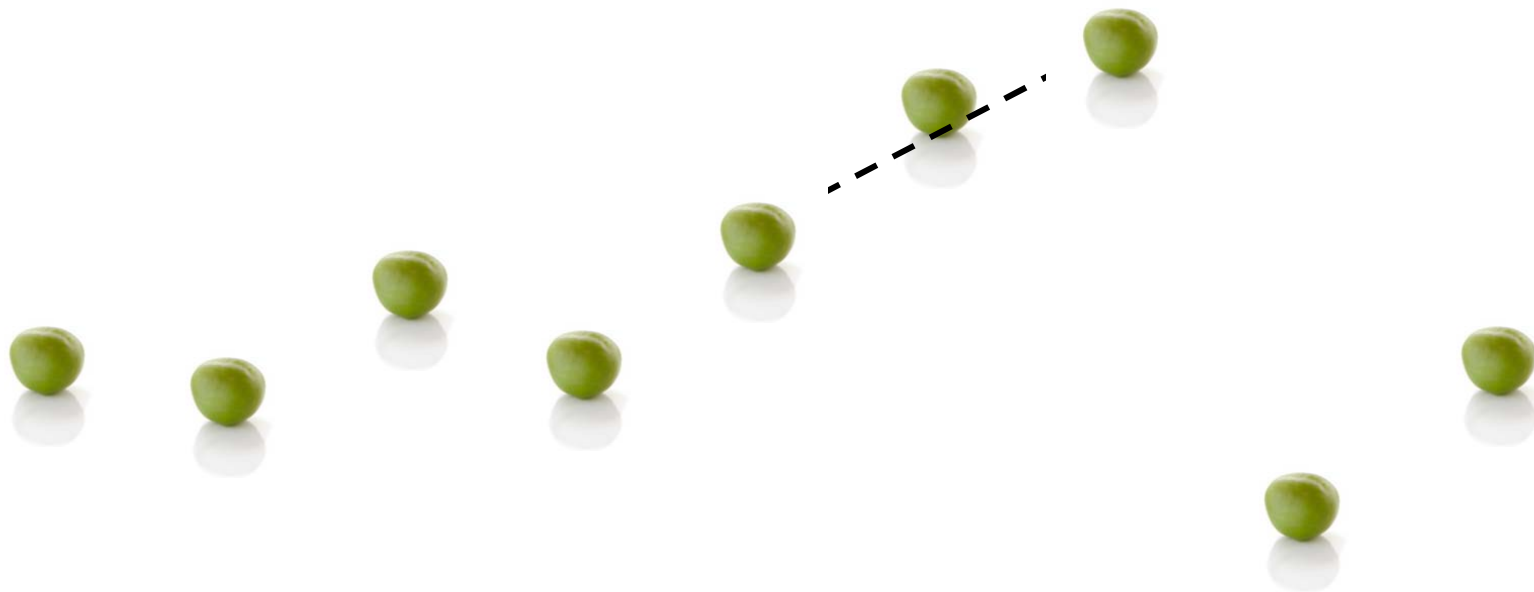
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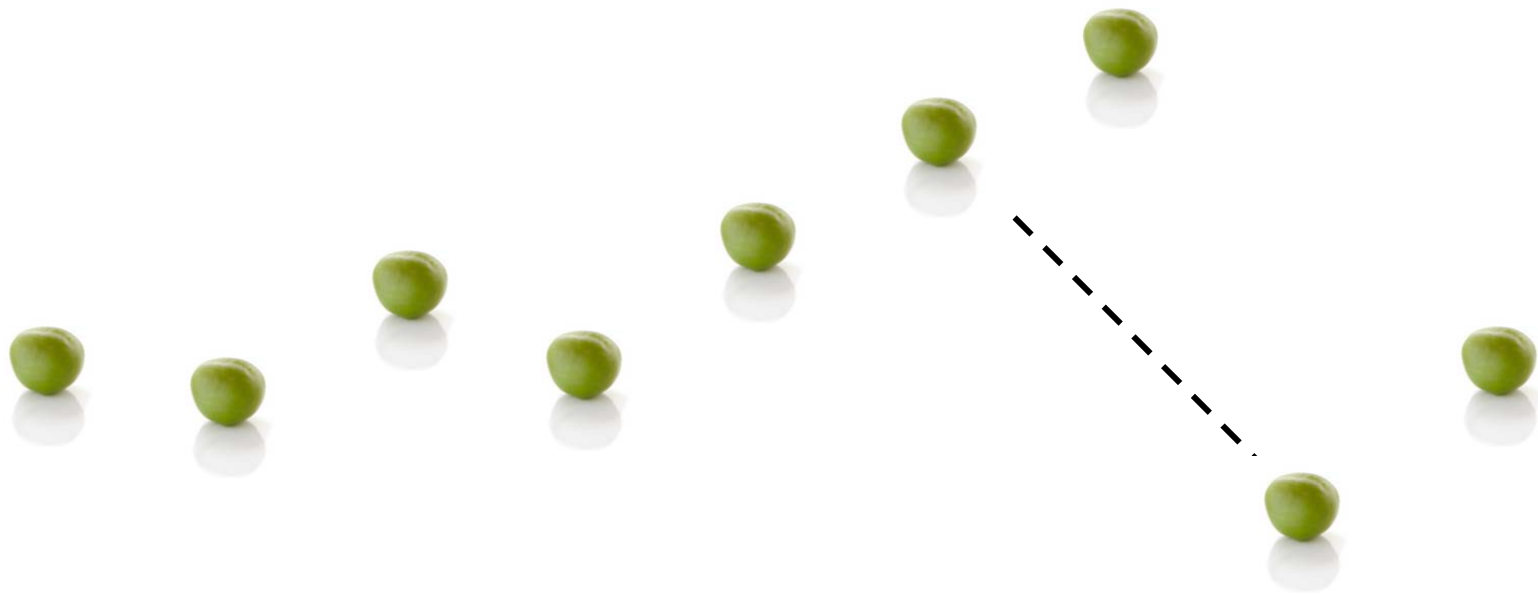
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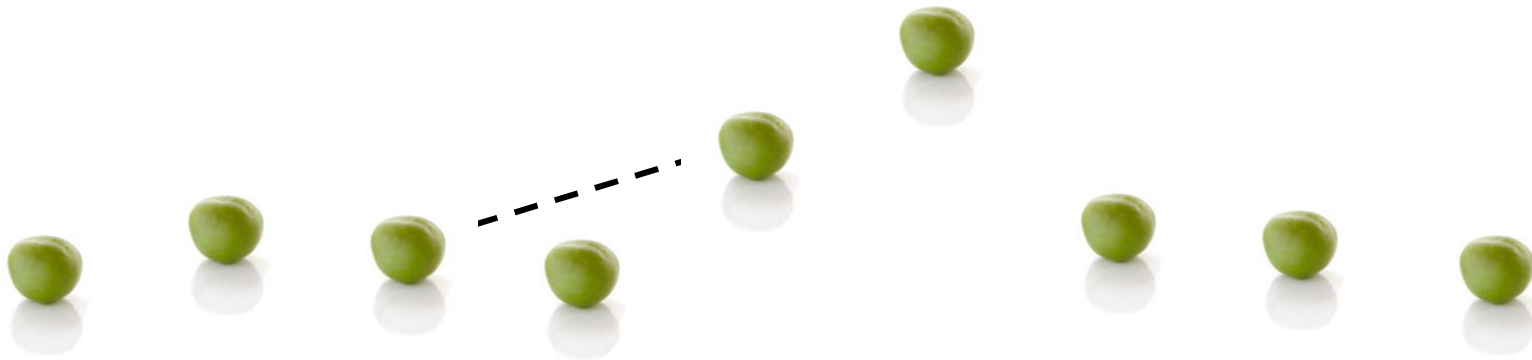
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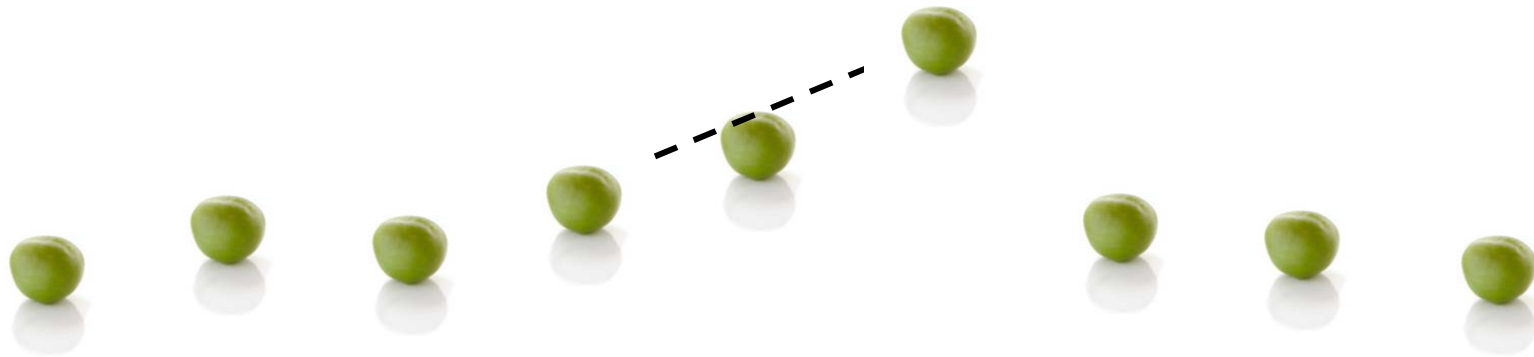
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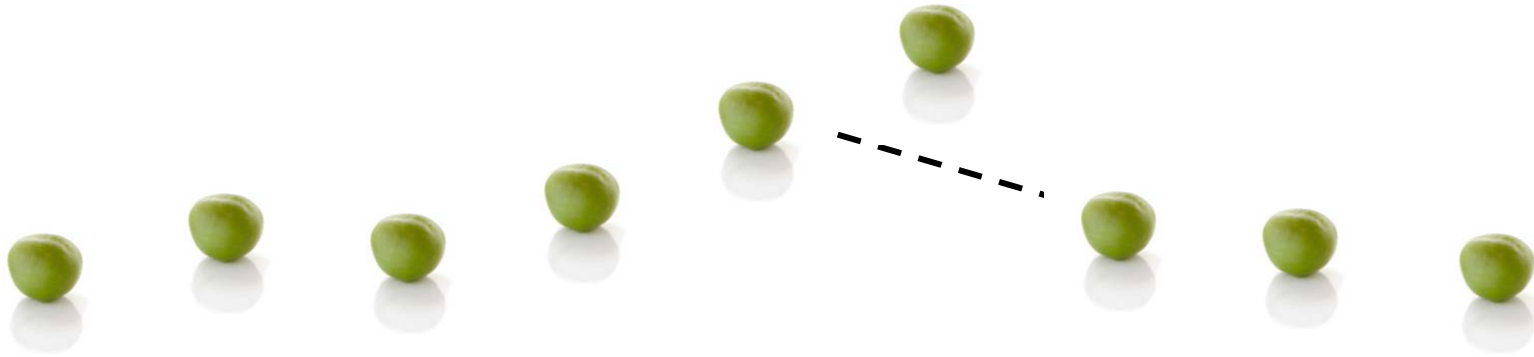
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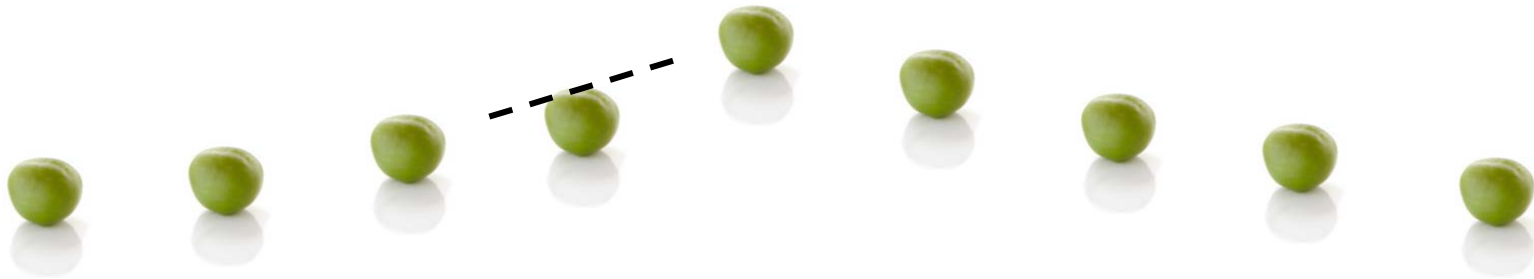
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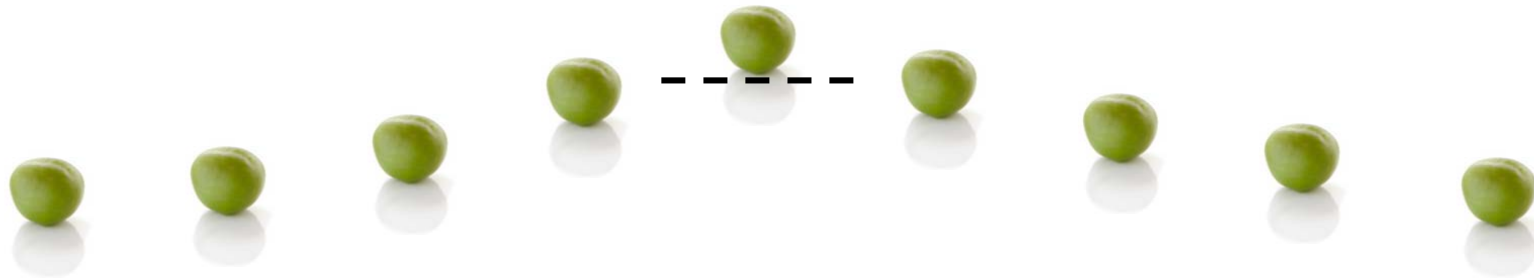
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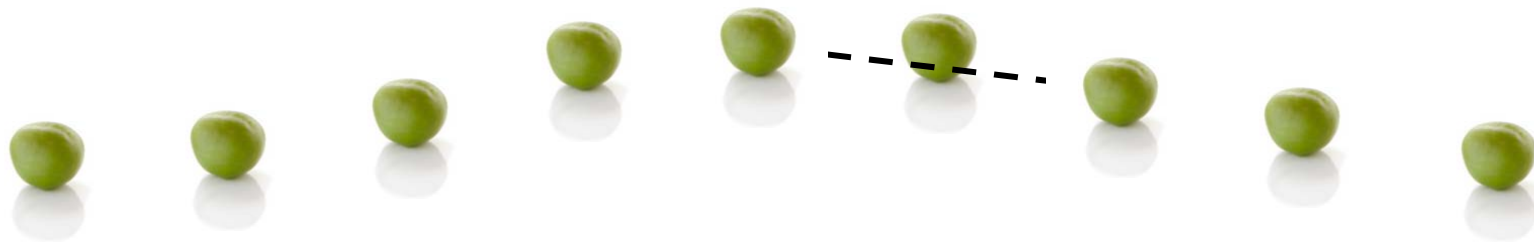
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Multigrid Methods – Presmoothing

Gauss Seidel



Multigrid Methods – Residual

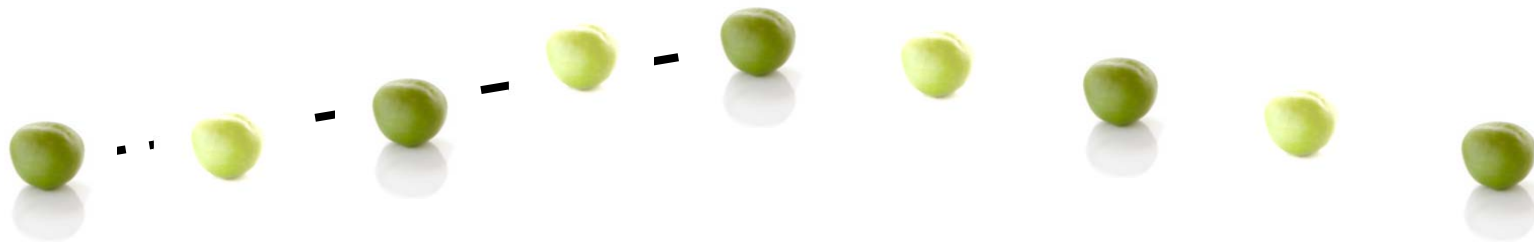
Almost zero \rightarrow neglected in following slides



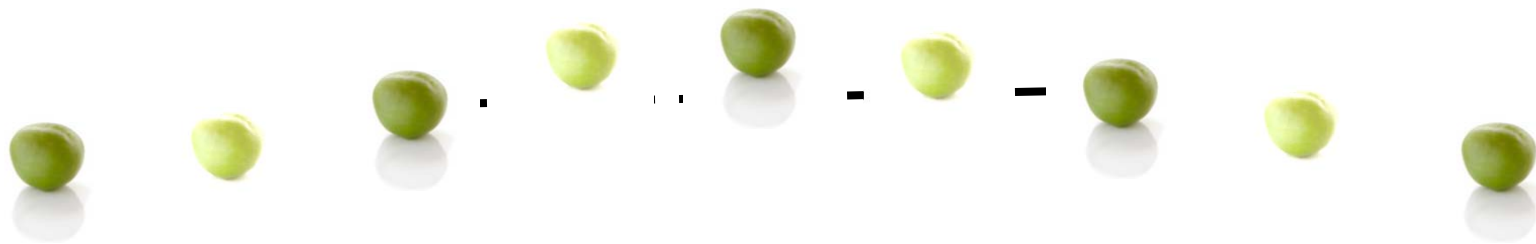
Multigrid Methods – Restriction



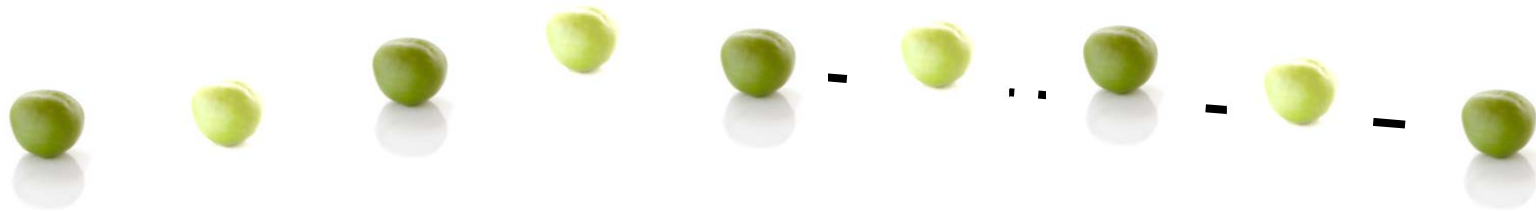
Multigrid Methods – Coarse Grid



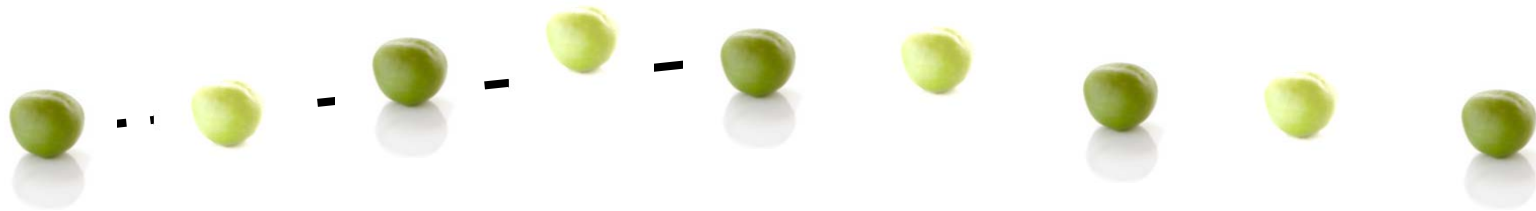
Multigrid Methods – Coarse Grid



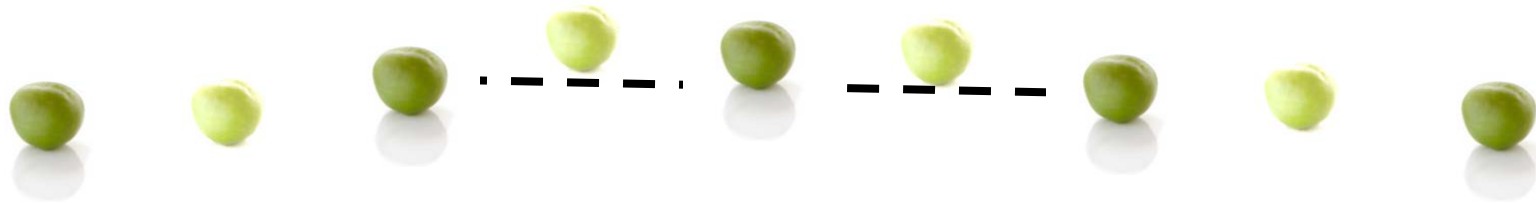
Multigrid Methods – Coarse Grid



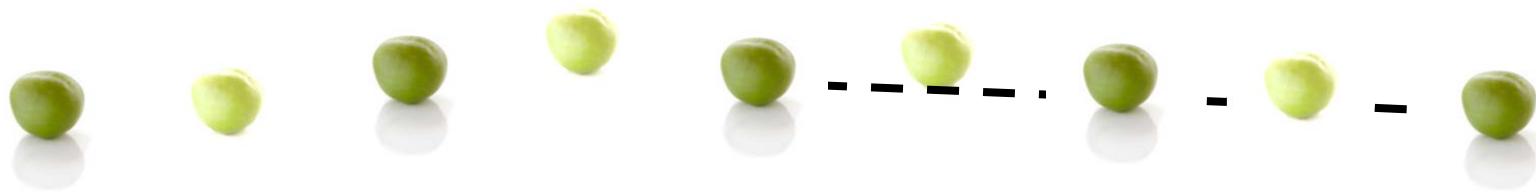
Multigrid Methods – Coarse Grid



Multigrid Methods – Coarse Grid



Multigrid Methods – Coarse Grid



Multigrid Methods – Coarse Grid



Multigrid Methods – Coarse Grid



Multigrid Methods – Coarse Grid



Multigrid Methods – Coarsest Grid



Multigrid Methods – Coarsest Grid



Multigrid Methods – Coarse Grid



Multigrid Methods – Coarse Grid



Multigrid Methods – Postsmoothing



Multigrid Methods – Postsmoothing



Multigrid Methods – Postsmoothing



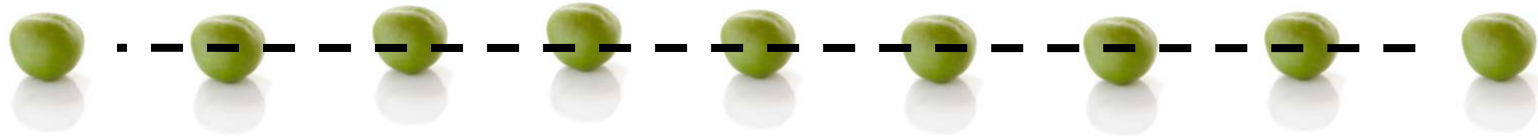
Multigrid Methods – Postsmoothing



Multigrid Methods – Postsmoothing

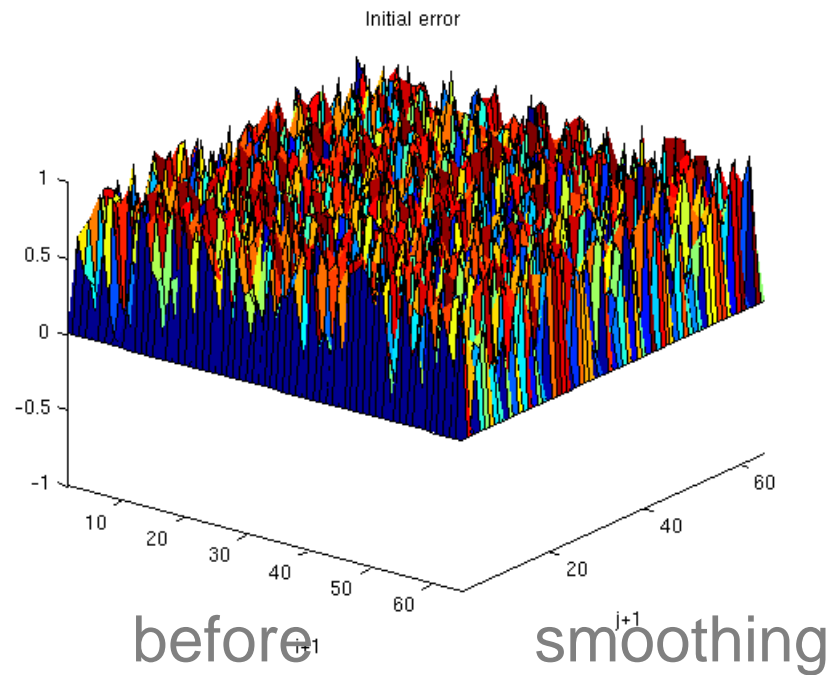


Multigrid Methods

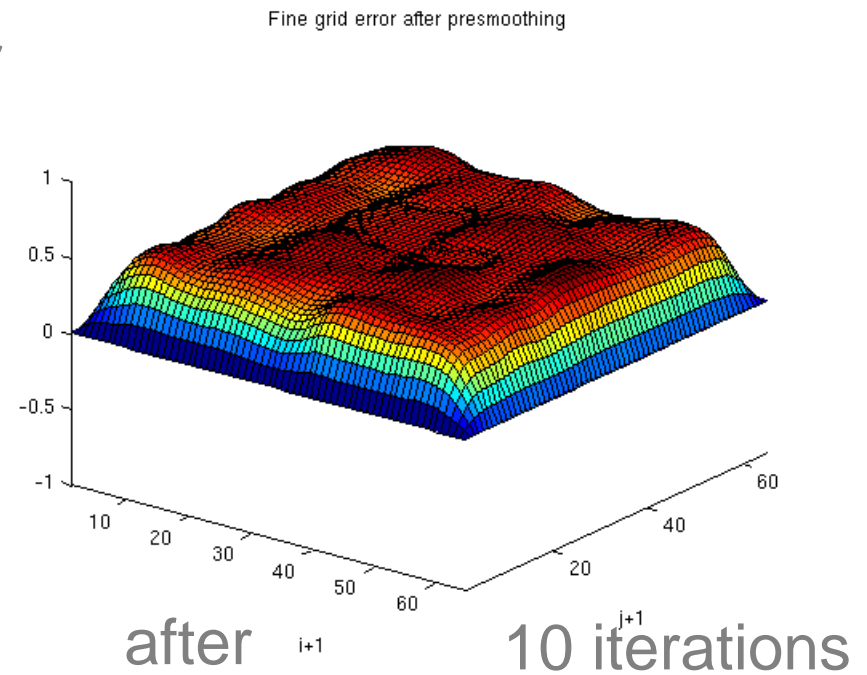


Multigrid

- remember: Gauss Seidel

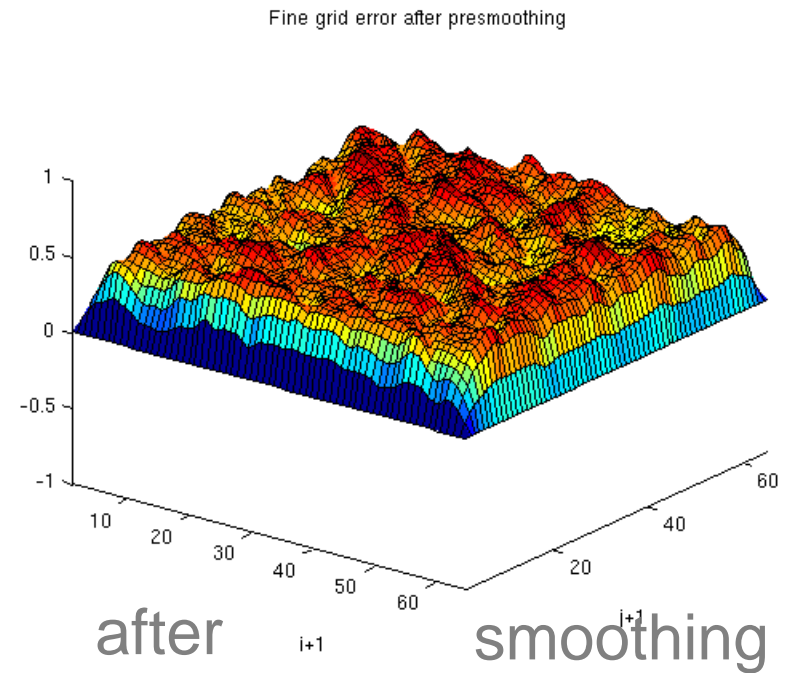
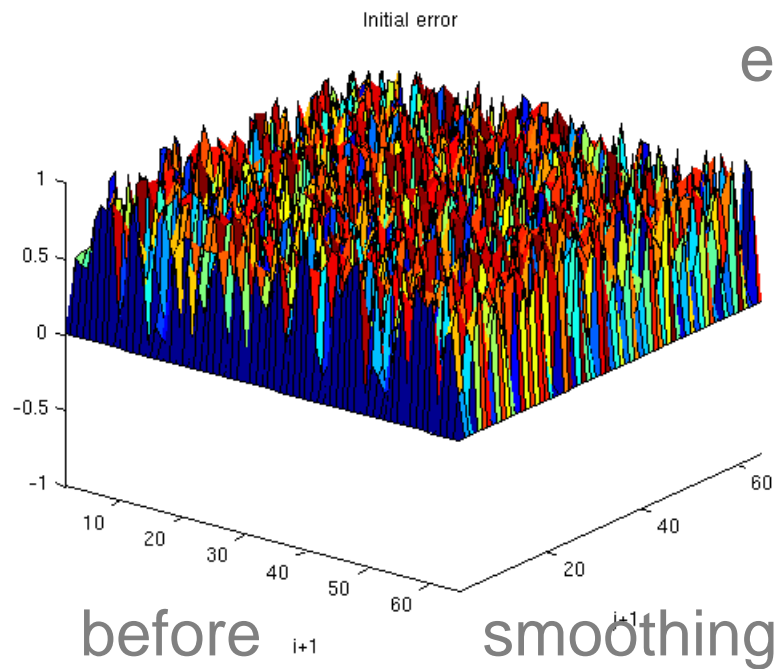


error



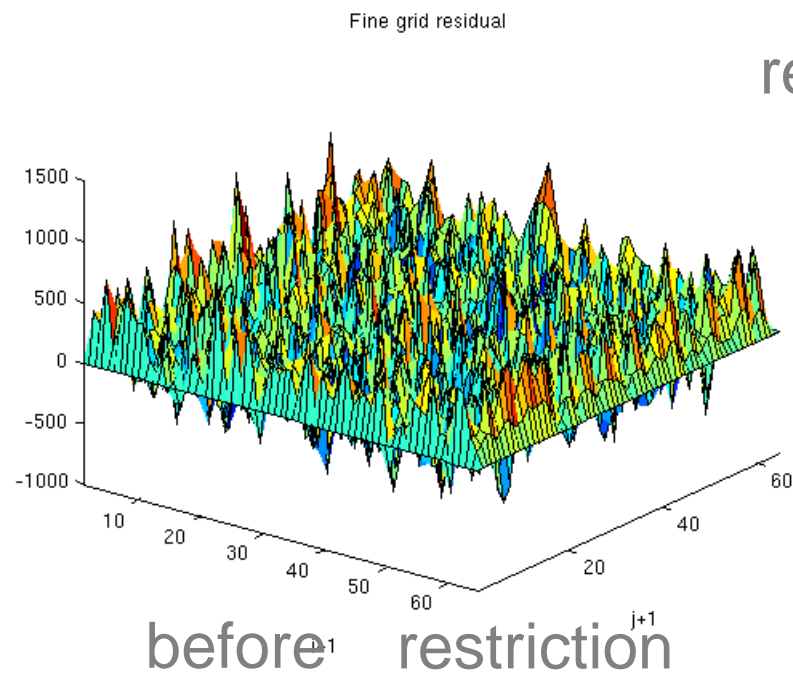
Multigrid

- fine grid
 - reduce high frequencies

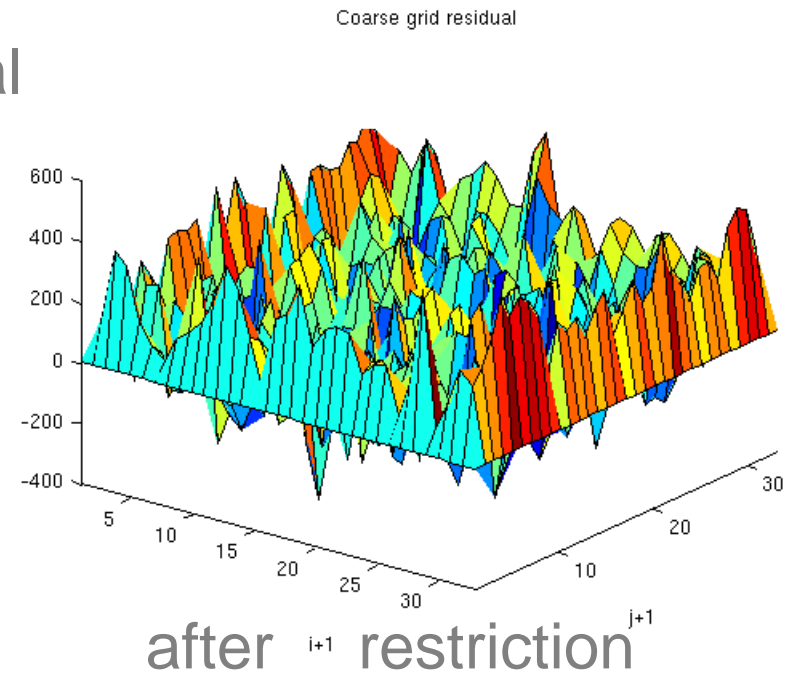


Multigrid

- switch to coarse grid
 - restrict residual

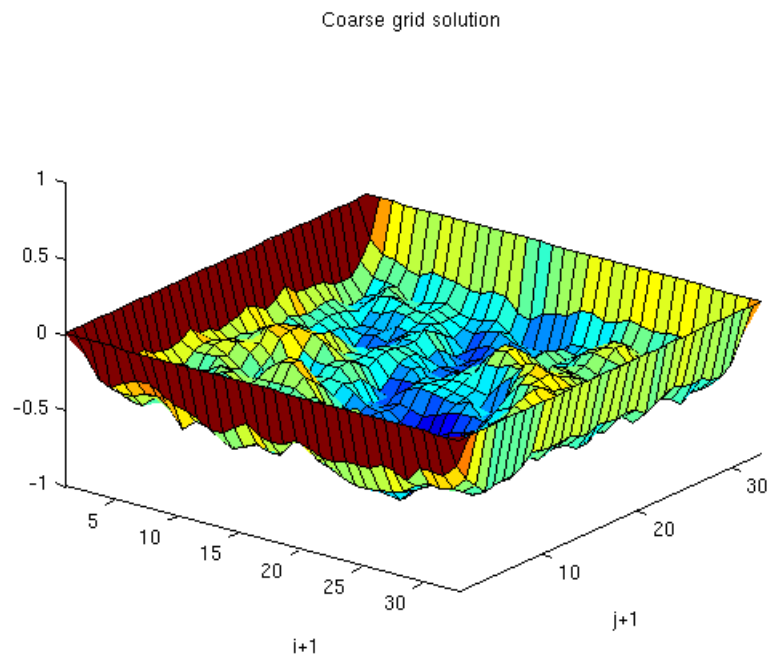


residual



Multigrid

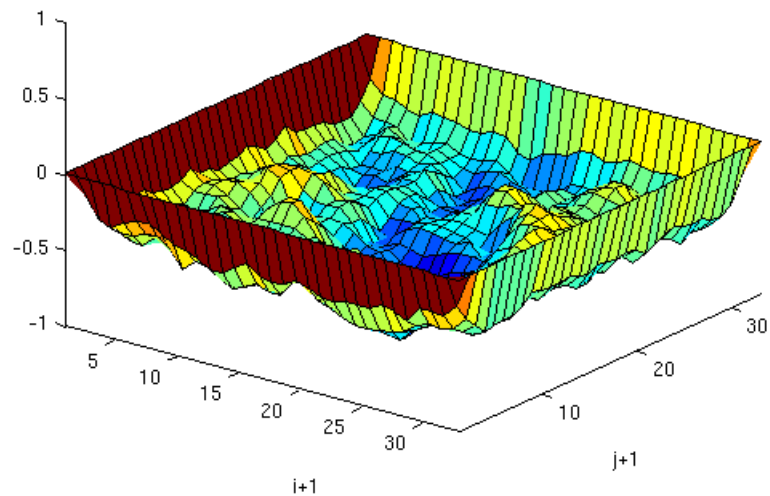
- solve coarse grid equation
 - recursive call of multigrid



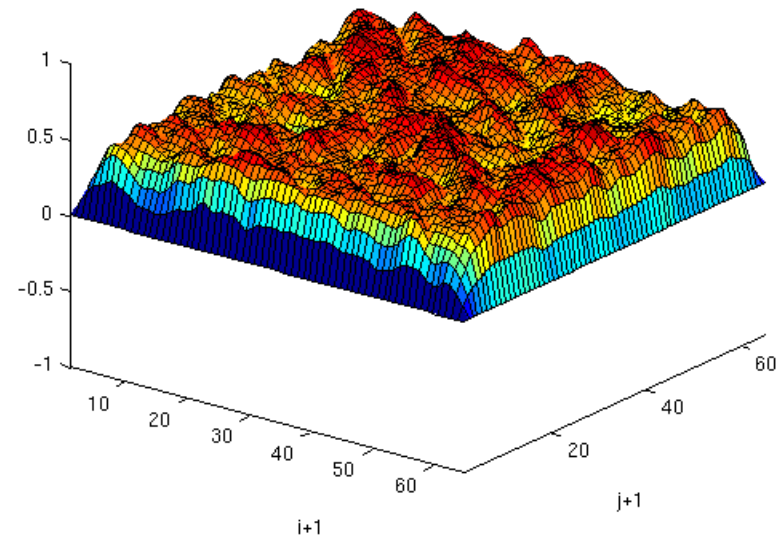
Multigrid

- solve coarse grid equation
 - recursive call of multigrid

coarse grid solution



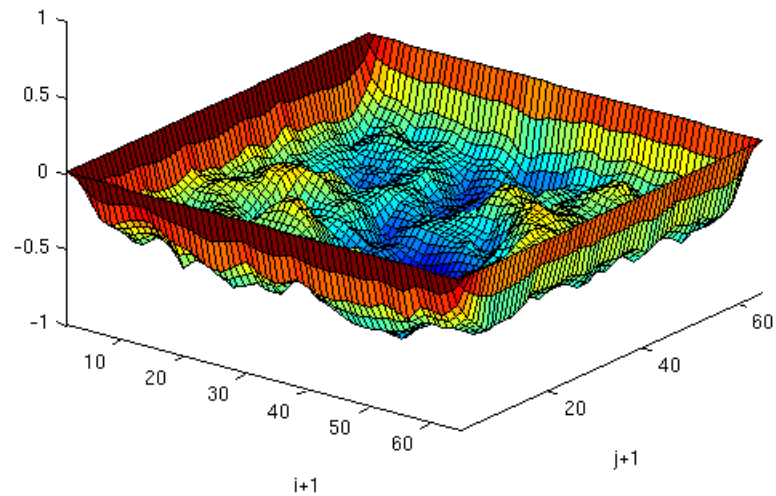
fine grid error



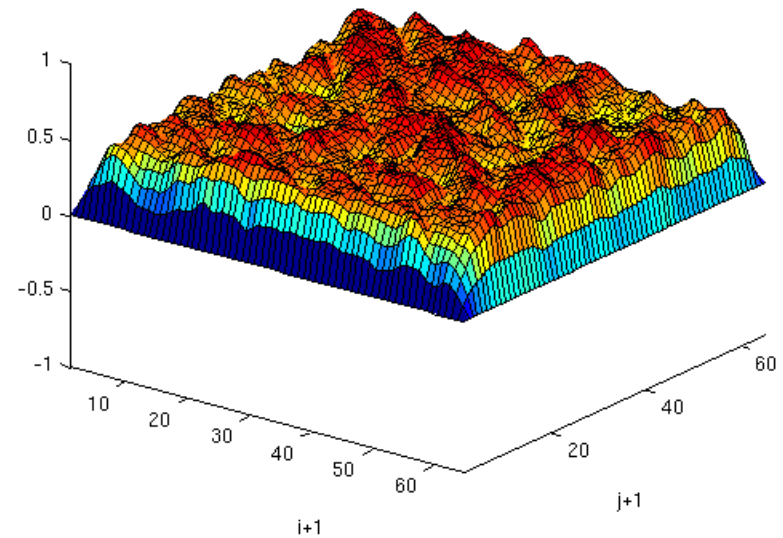
Multigrid

- switch to fine grid
 - interpolate coarse grid solution

interpolated coarse grid solution



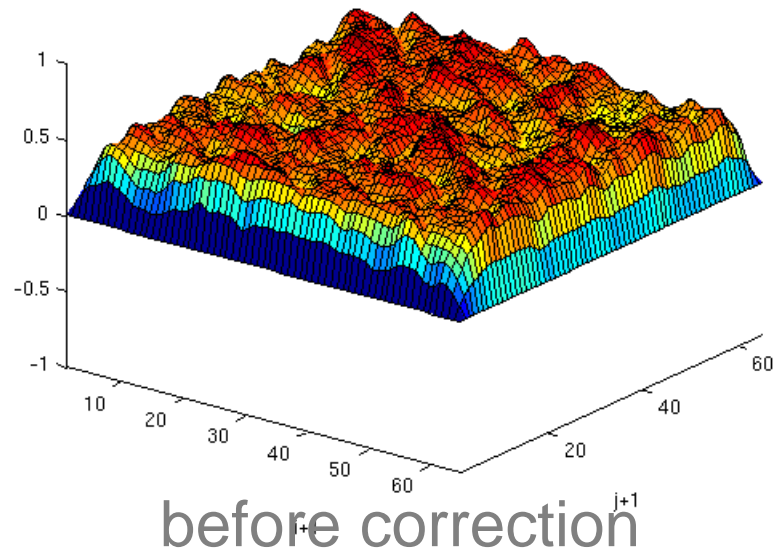
fine grid error



Multigrid

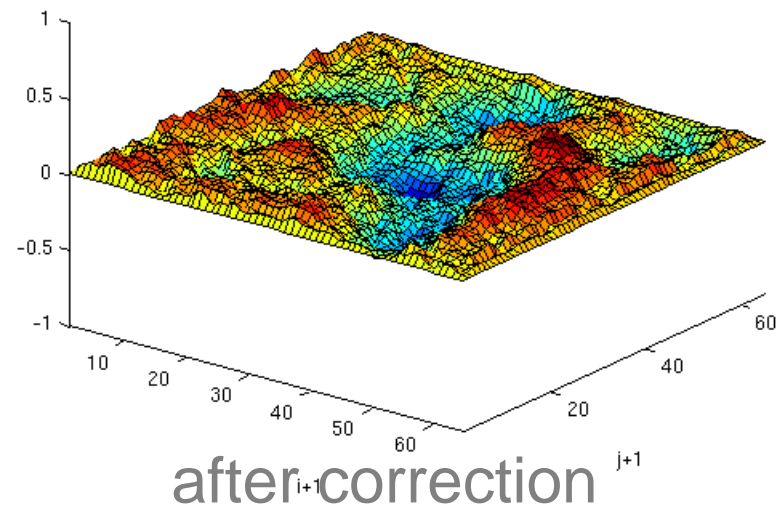
- switch to fine grid
 - apply coarse grid correction

Fine grid error after presmoothing



fine grid error

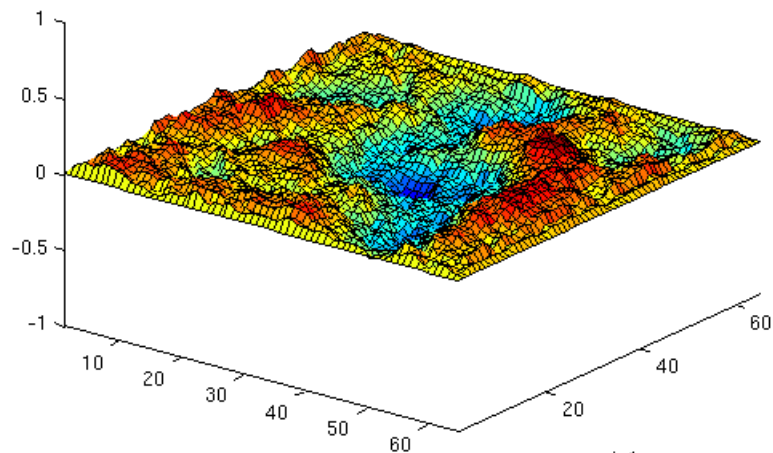
Fine grid error after correction



Multigrid

- fine grid
 - eliminate new high frequencies

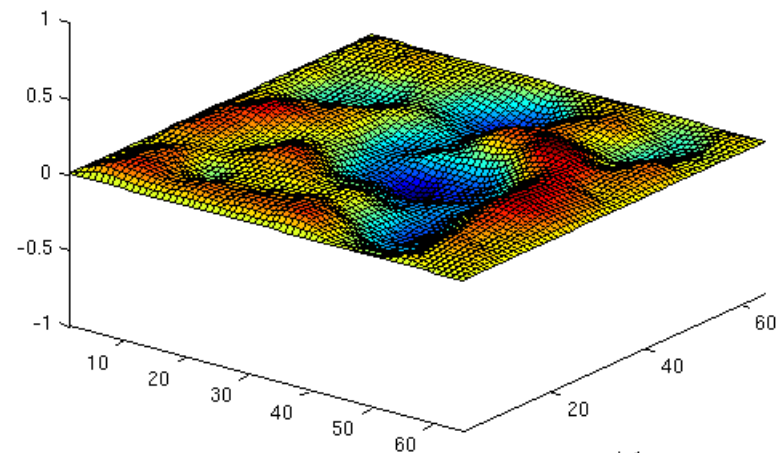
Fine grid error after correction



before smoothing^{j+1}

fine grid error

Fine grid error after postsmoothing

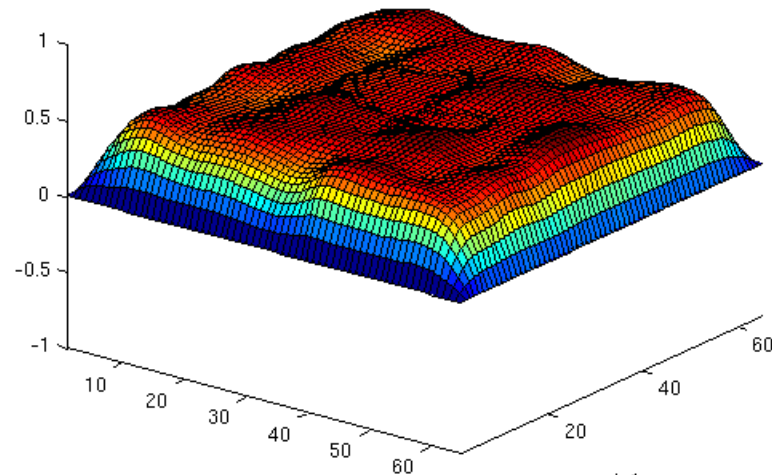


after smoothing^{j+1}

Multigrid

- comparison Gauss-Seidel – multigrid

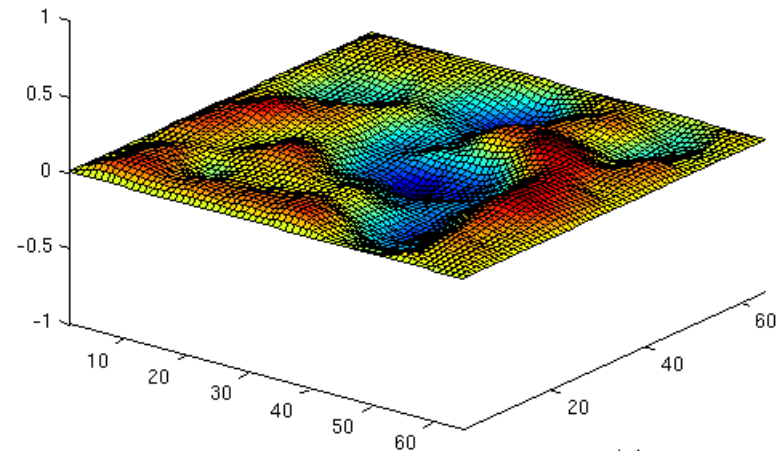
Fine grid error after presmoothing



after 10 Gauss-Seidel iterations ^{$i+1$}

error

Fine grid error after postsmoothing



after 1 multigrid iteration ^{$i+1$}

Multigrid – Cycles

- V-cycle: one recursive call
- W-cycle: two recursive calls
- F-cycle: V-cycle on each level