A Dimension-adaptive Sparse Pseudo-Spectral Projection Method in Linear Gyrokinetics

A.1 Plasma micro-turbulence

- goal: characterization of the turbulent transport in magnetically confined fusion devices by gyrokinetic fully nonlinear simulations
- modeling: 5D integro-differential Vlasov-Maxwell system of eqs.
- here: restriction to linear physics
- less complicated testbed
- insights into sensitivities of underlying micro-instabilities

Linear/local simulations

- here: two particle species: deuterium ions and electrons
- typical output of interest: dominant eigenmode with magnitude and frequency

Global gyrokinetic simulation of turbulence

B.1 Sparse pseudo-spectral projection

- sparse approximation based on pseudo-spectral projection operators
- constructed on internal aliasing error-free spaces
- starting point: 1D projection operators $F_1^{(1)}(\tilde{f}(\theta)) = \sum_{j=0}^{2} p_j f_j(\theta)$

\[
\left( F_1^{(1)}(\tilde{f}(\theta)) \rightarrow F_2^{(1)}(\tilde{f}(\theta)) \right) = \sum_{j=0}^{2} p_j \tilde{f}_j(\theta)
\]

- internal aliasing error-free construction: $(\varphi_k, \varphi_j) = \rho_0(\varphi_k, \varphi_j), \forall k, j$

- in $d$-dimensions: $i = (i_1, \ldots, i_d) \in \mathbb{N}^d, \Delta_l^{(i)} = F_2^{(1)}(\tilde{f}(\theta)) - F_1^{(1)}(\tilde{f}(\theta))$

\[
\Delta_l^{(i)}(\theta) = \sum_{k \in \mathbb{N}^d} \Delta_k^{(i)}(\theta)
\]

- construct $C_2^d$ adaptively using a posteriori heuristics

Interpolation-to-spectral-projection mapping

- alternative approach: interpolation instead of spectral projection
- for each subspace, find the equivalent spectral coefficients

C.1 Benchmark test case 2: 8 stochastic parameters

- simple geometry
- the dominant mode clearly changes with the wave number $k_{\phi, A}$

<table>
<thead>
<tr>
<th>stochastic parameter</th>
<th>symbol</th>
<th>left bound</th>
<th>right bound</th>
<th>output of interest (growth rate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>plasma beta</td>
<td>$\rho$</td>
<td>0.96e-04</td>
<td>9.6e-04</td>
<td></td>
</tr>
<tr>
<td>collision frequency</td>
<td>$\nu_c$</td>
<td>0.26e-02</td>
<td>3.23e-02</td>
<td></td>
</tr>
<tr>
<td>magnetic shear</td>
<td>$\mu$</td>
<td>0.7310 0.820</td>
<td></td>
<td></td>
</tr>
<tr>
<td>safety factor</td>
<td>$q$</td>
<td>1.100</td>
<td>1.470</td>
<td></td>
</tr>
<tr>
<td>density gradient</td>
<td>$\gamma$</td>
<td>1.667 2.77</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ions temperature gradient</td>
<td>$T$</td>
<td>7.000 12.500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ions temperature</td>
<td>$T$</td>
<td>0.950 1.070</td>
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<td></td>
</tr>
<tr>
<td>electrons temperature</td>
<td>$T$</td>
<td>7.000 12.500</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- dimension-adaptivity ($D_{adapt} = 12$) based on:

\[
= \text{error/cost + diagonal derivative}
\]

\[
= \text{error/cost} + \text{diagonal derivative}
\]

C.2 Real-world problem: 3 stochastic parameters

- complex geometry
- based on experimental data

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<tr>
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<th>right bound</th>
<th>output of interest (growth rate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>density gradient</td>
<td>$\gamma$</td>
<td>1.156 1.924</td>
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<td>ions temperature gradient</td>
<td>$T$</td>
<td>2.016 3.494</td>
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<tr>
<td>electrons temperature gradient</td>
<td>$T$</td>
<td>4.916 6.716</td>
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<td></td>
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</tbody>
</table>

- construct the sparse grid surrogate using two approaches:

\[
= \text{pseudo-spectral projection}
\]

\[
= \text{interpolation + interpolation-to-spectral-projection mapping}
\]

- dimension-adaptivity ($D_{adapt} = 30$) based on:

\[
= \text{error/cost criterion with tol = 5 \cdot 10^{-3}}
\]

\[
= \text{error/cost + diagonal derivative with tol = 5 \cdot 10^{-3} and tol^2 = 5 \cdot 10^{-5}}
\]

- conclusion: interpolation + adaptivity based on directional variance clearly superior

References