Enhanced divergence-free elements for efficient incompressible flow simulations in the PDE framework Peano

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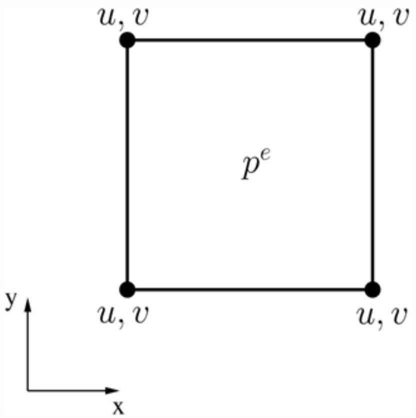
Outline

- Derivation of Div-free Ansatz Functions
- Enhanced Div-free Elements
- Numerical Results
  - Checkerboard Driven Cavity
  - Driven Cavity
  - Flow around a Cylinder
- Outlook

Derivation of Div-free Ansatz Functions

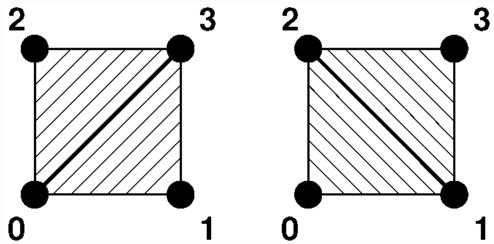
- Incompressible Navier-Stokes Equations

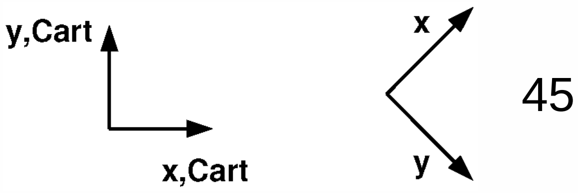
$$\begin{aligned} \nabla \cdot \mathbf{u} &= 0 \\ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u} + \frac{1}{\rho}\nabla p - \nu\,\Delta \mathbf{u} &= \mathbf{0} \end{aligned}$$



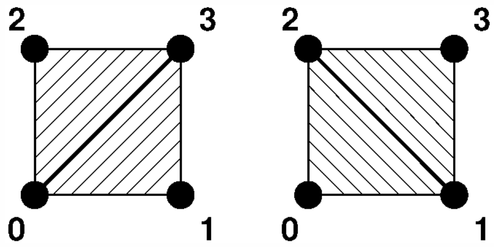
- Discretisation
  - low-order FEM (Q1Q0, etc.)
- Divergence-free elements:
  - solenoidal velocities in **every** point in a cell
  - simultaneous conservation of momentum **AND** energy

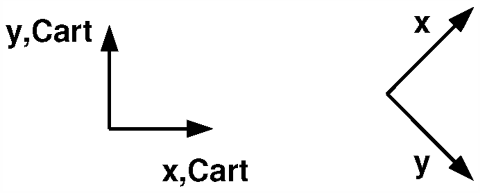
Derivation of Div-free Ansatz Functions





Derivation of Div-free Ansatz Functions





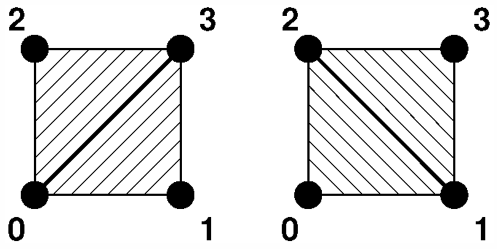
45

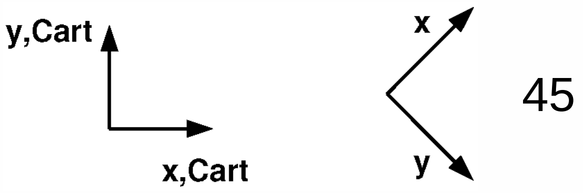
du/dx = const = u3 - u0

dv/dy = const = v1 - v2

div(u) = u3 - u0 + v1 - v2 = const = 0

Derivation of Div-free Ansatz Functions





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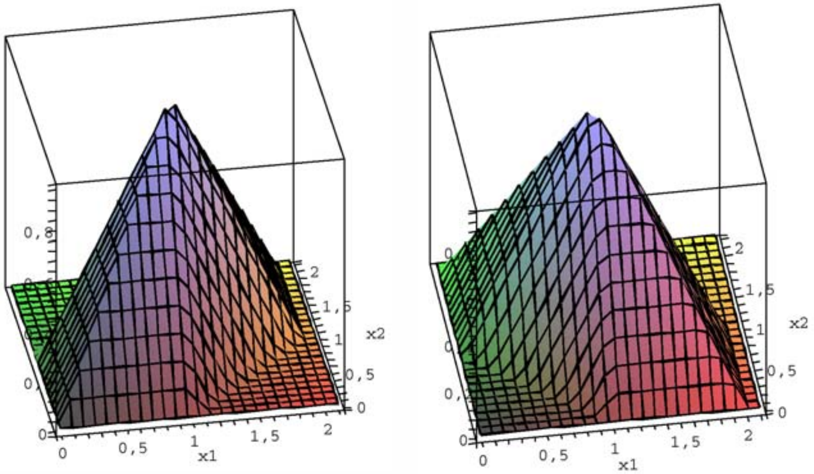
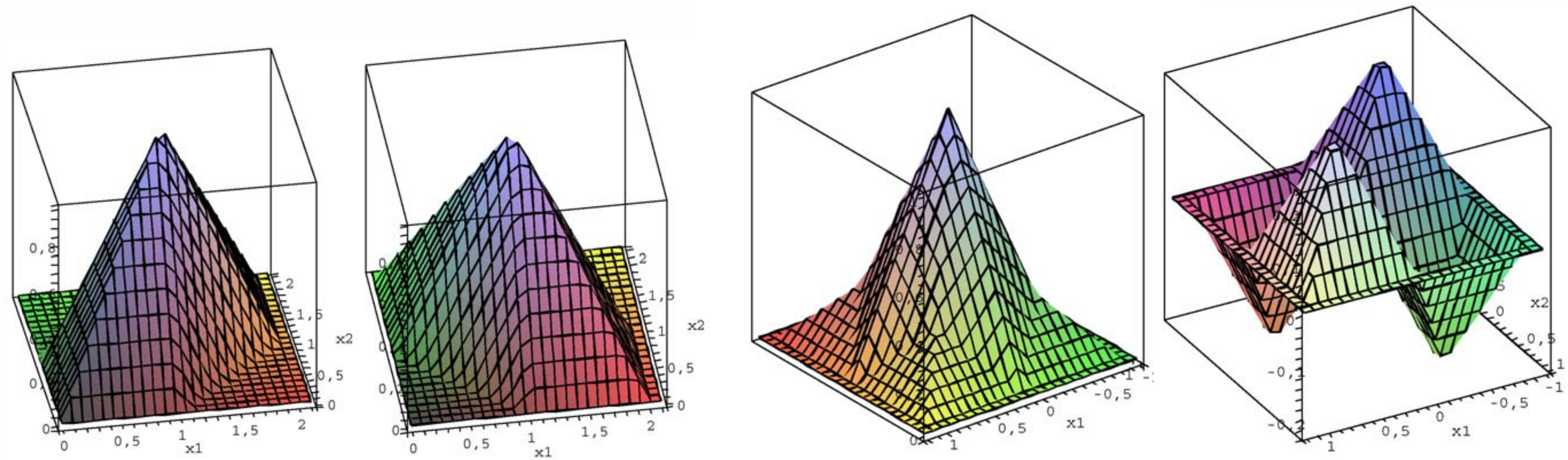


Derivation of Div-free Ansatz Functions

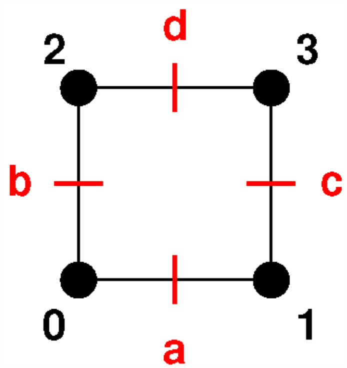
- Ansatz functions 45 and Cartesian:

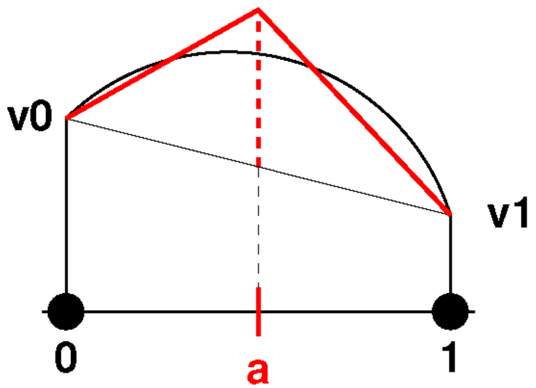


- Advantages of 45 representation:
  - Simplified derivation and representation of elements
  - Performance: ~20% less runtime for evaluation of operators D and C

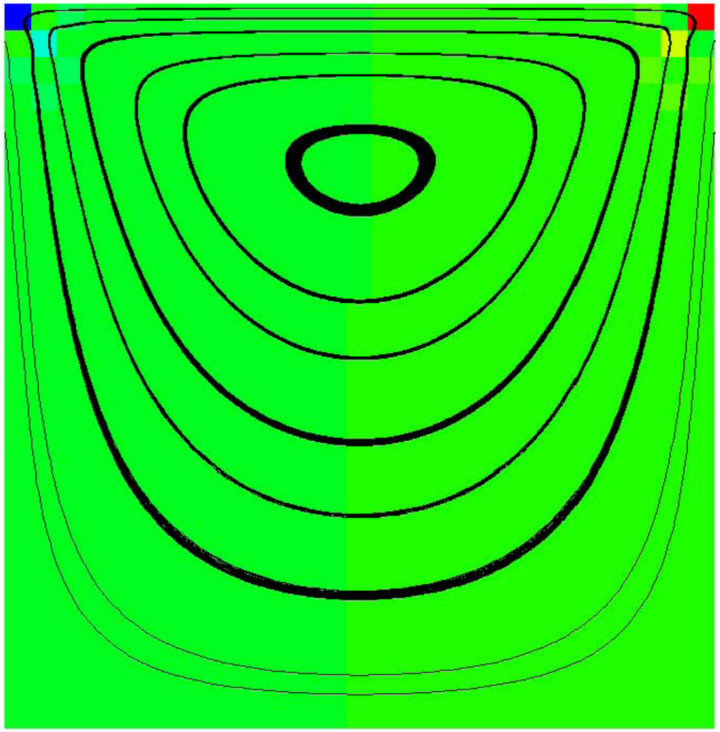
Enhanced Div-free Elements

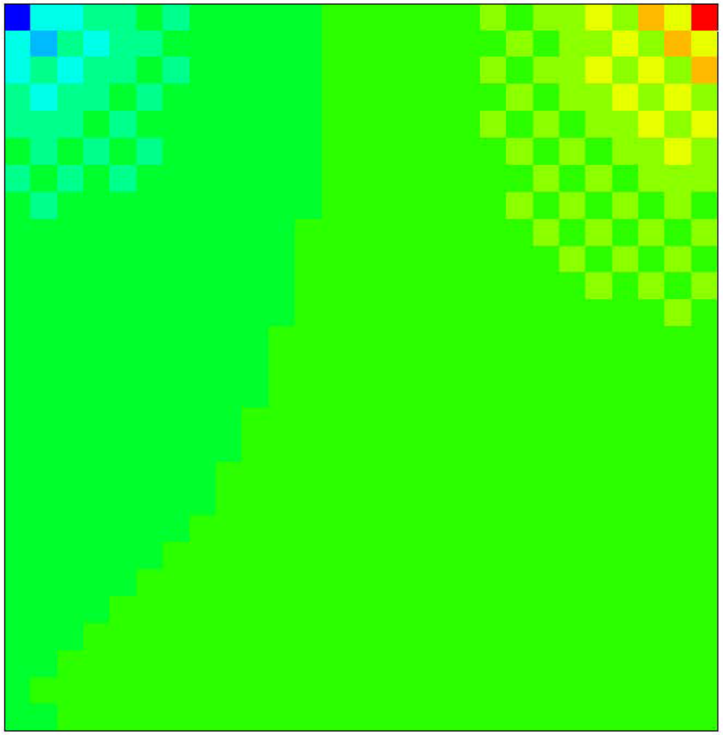
- Additional DoF on faces:
  - exact representation of fluxes on edges
  - no checkerboarding





Numerical Results – Checkerboard Driven Cavity



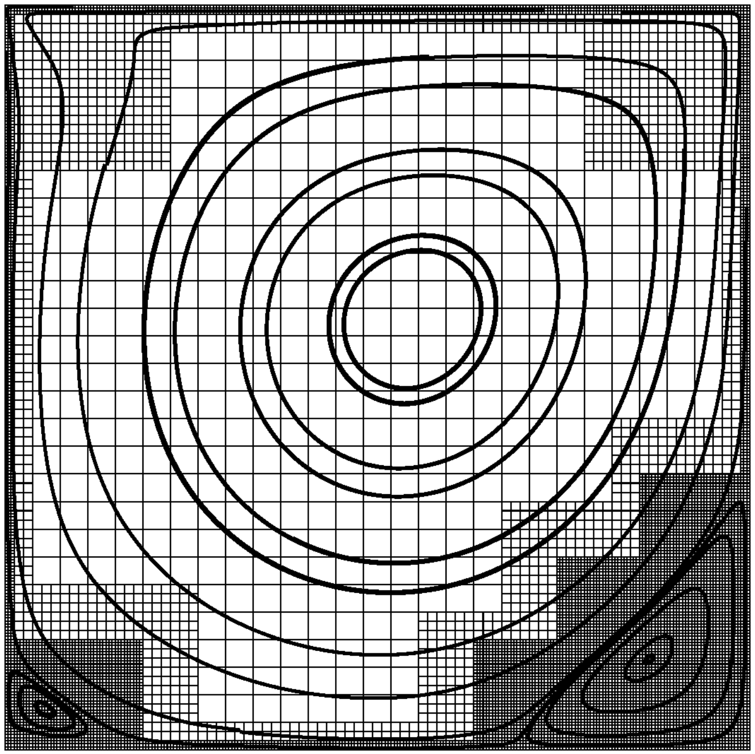


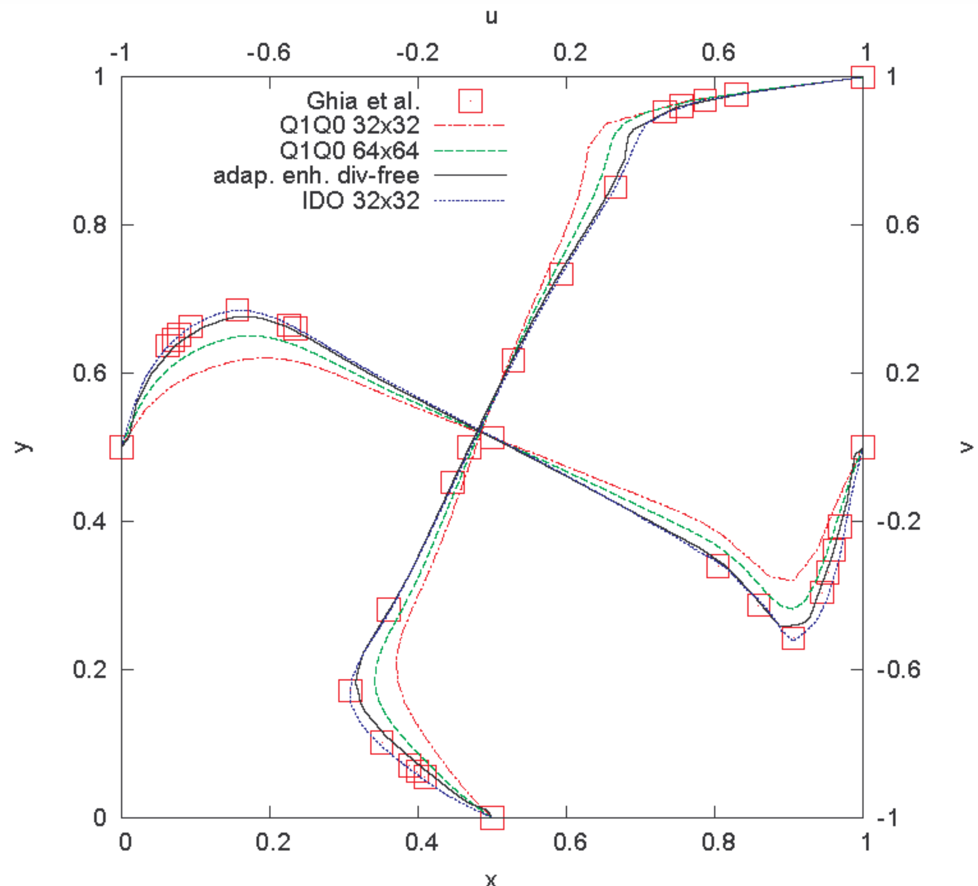
enhanced div-free
(steady state)

Re=1

Q1Q0
(step1, no convergence!)

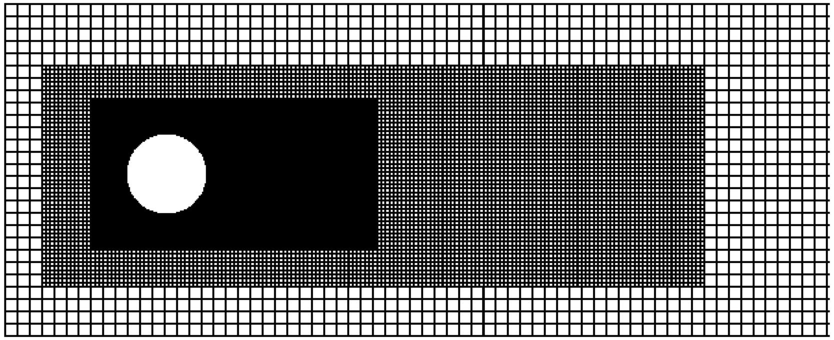
Numerical Results – Driven Cavity



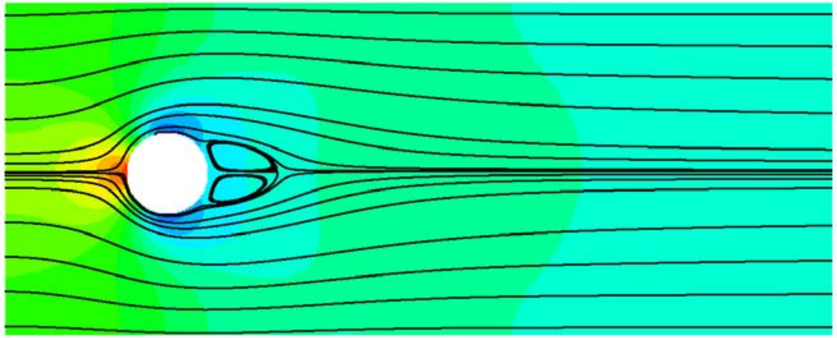


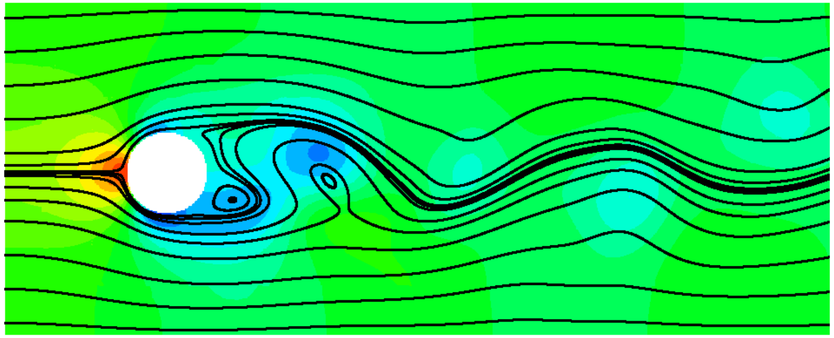
Re=1000

Numerical Results – Flow around a Cylinder



# DoF	Re = 20		Re = 100		
	c_d	c_l	$C_{d,max}$	$C_{l,max}$	St
88,857	5.68	0.0151	3.225	0.94	0.299
ref.	5.58	0.0107	3.230	1.00	0.298





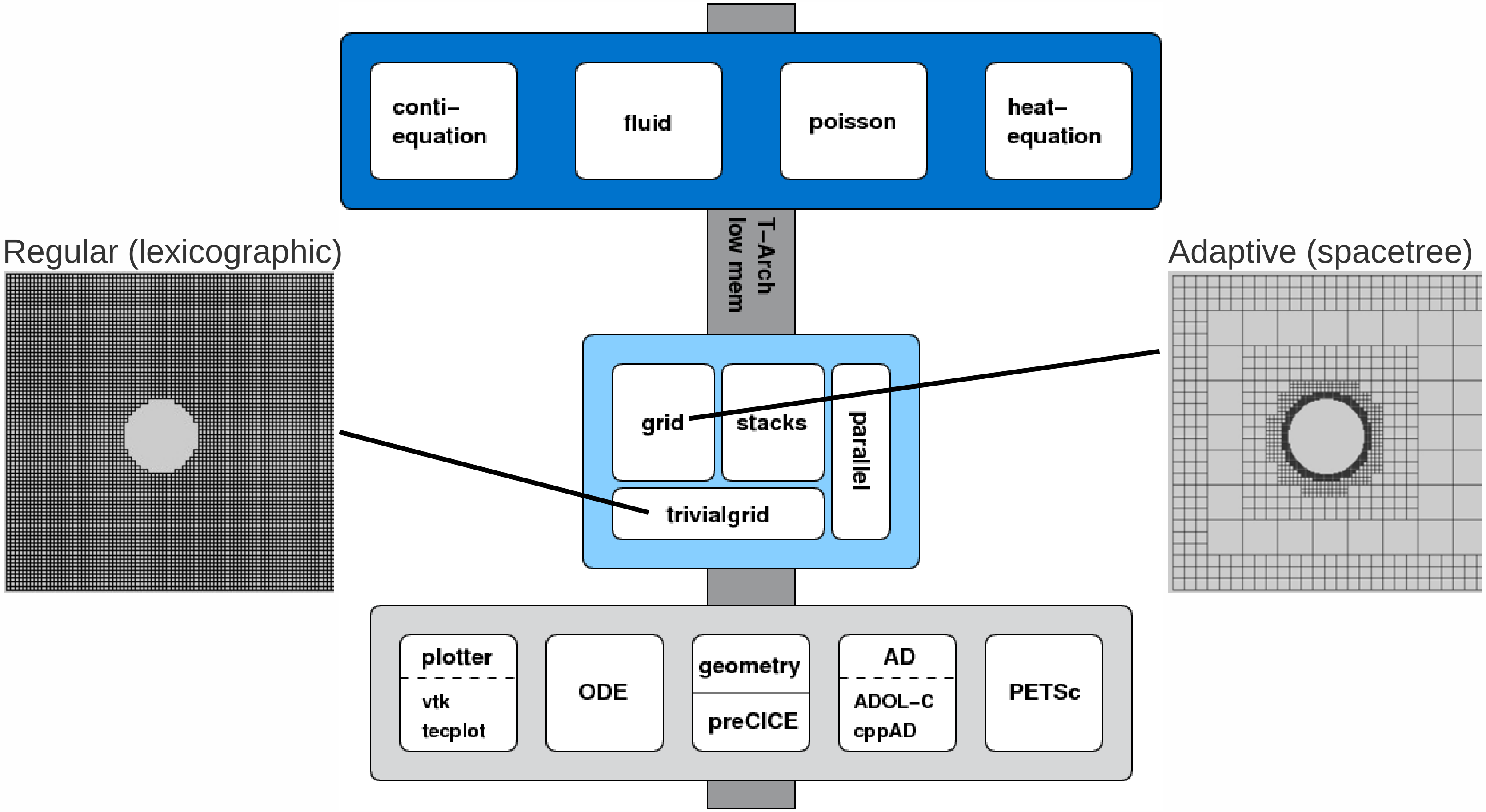
Outlook

- **Multigrid**
  - Peano Framework designed for hierarchical applications
  - Speed up computations while keeping low memory requirements

- Extension of (enhanced) div-free elements to **3D**

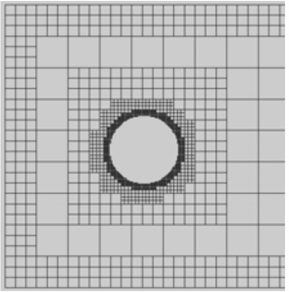
Thanks for your attention!

The PDE Framework Peano



http://www5.in.tum.de/peano/

The PDE Framework Peano

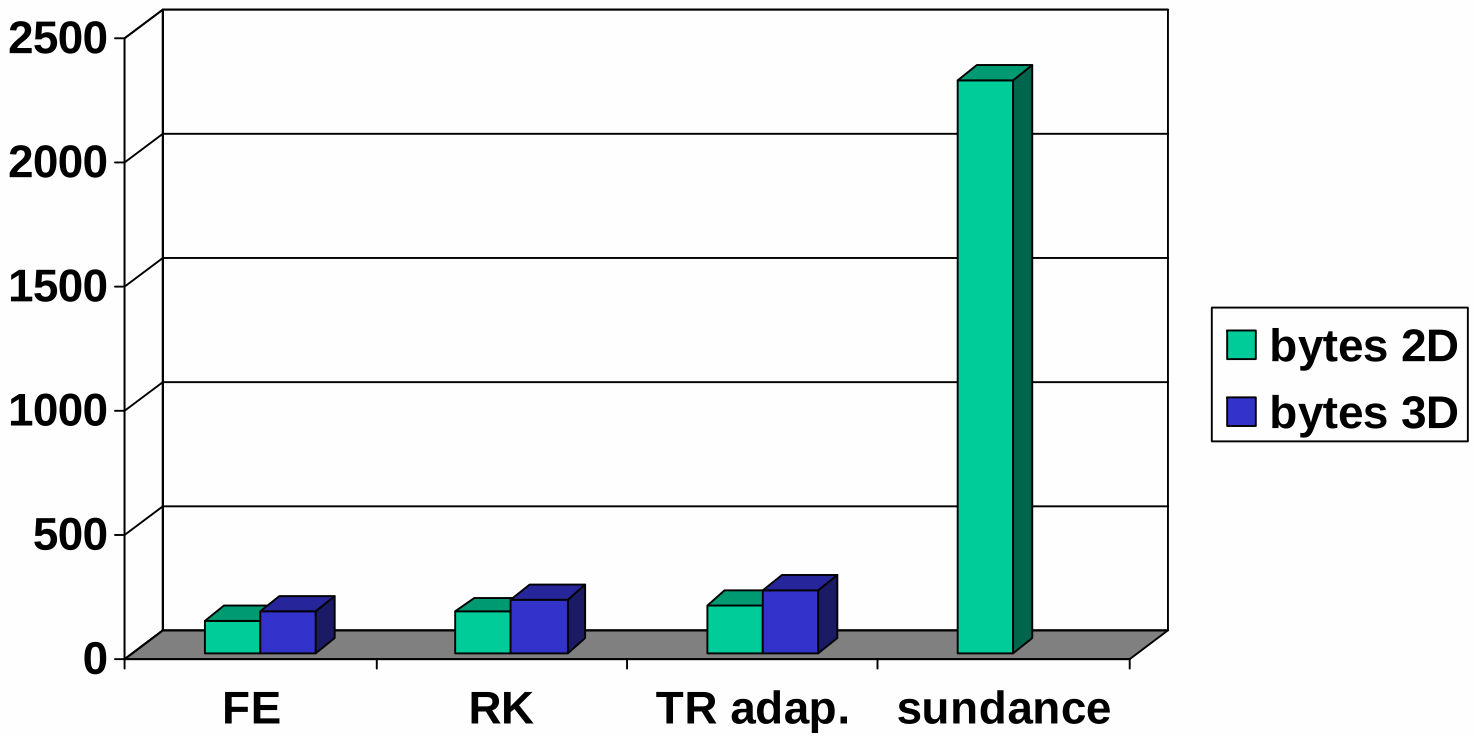


- Cartesian grids (arbitrary dimensions)
- Plug-in concept for applications
- Space-filling curves, spacetrees, and stack data structures
  - Strictly element-wise access
  - Low memory demands
  - Dynamical load balancing
  - Moving geometries, dynamical adaptivity, geometric multigrid
- Software Engineering
  - automatic tests, continuous integration, OO, design patterns, ...
- CFD component
  - Incompressible flow (FEM, IDO)
  - Explicit + implicit time-integration schemes (FE, RK4, BE, (adaptive) TR)

http://www5.in.tum.de/peano/

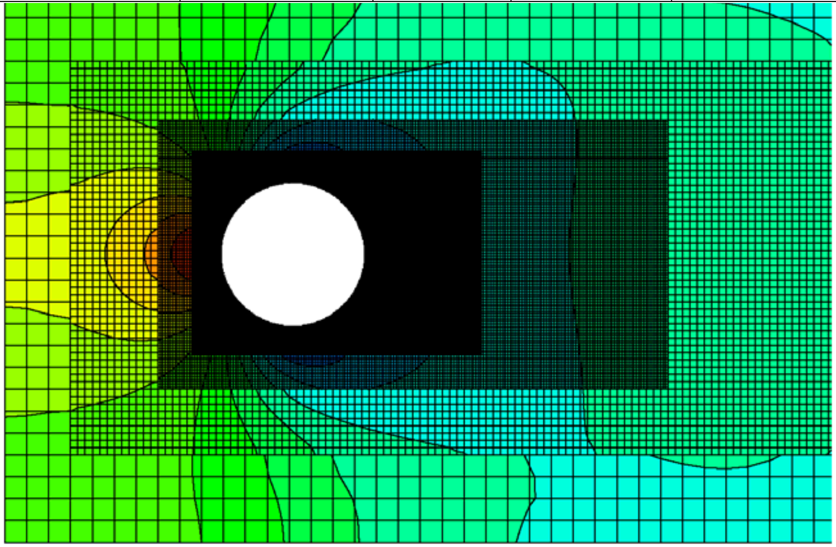
Backup I

Low memory requirements (FEM + adap.):



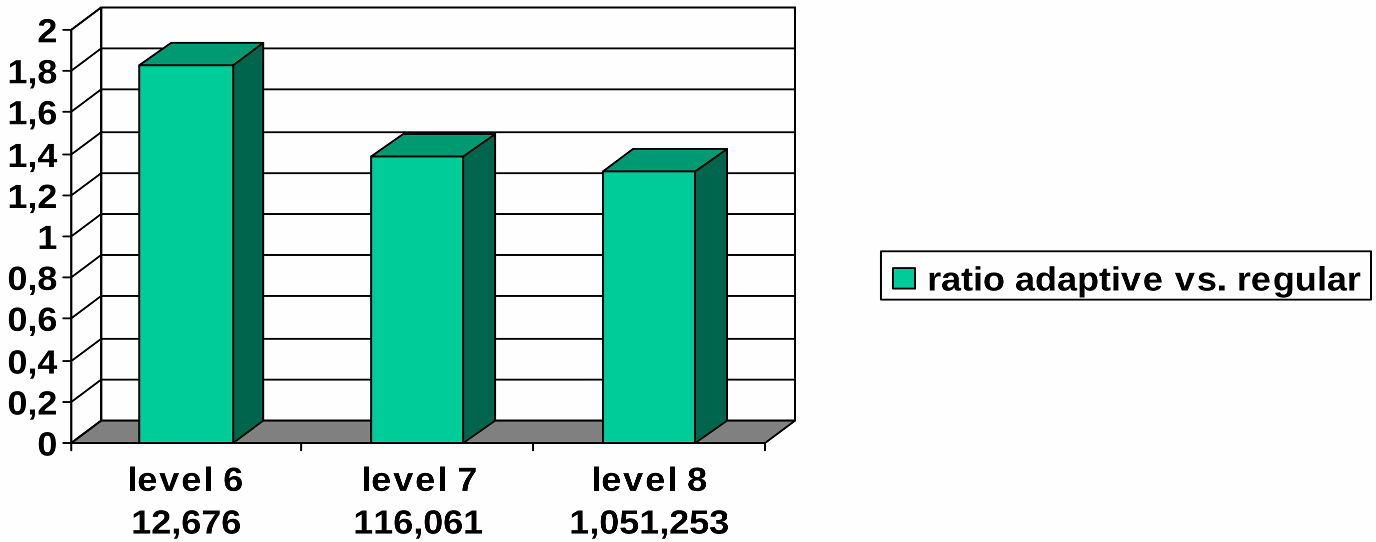
Numerical Results – FEM Q1Q0

max. level	min. level	#total DoF	c_d	c_l	CPU time per time step
8	6 (box)	88857	5.680	0.0150	0.71
9	7	125041	5.591	0.0113	1.03
9	8	1057877	5.561	0.0112	9.46
9	6 (box)	261501	5.586	0.0115	2.46
ref. data		–	5.580	0.0107	–



Numerical Results - Performance

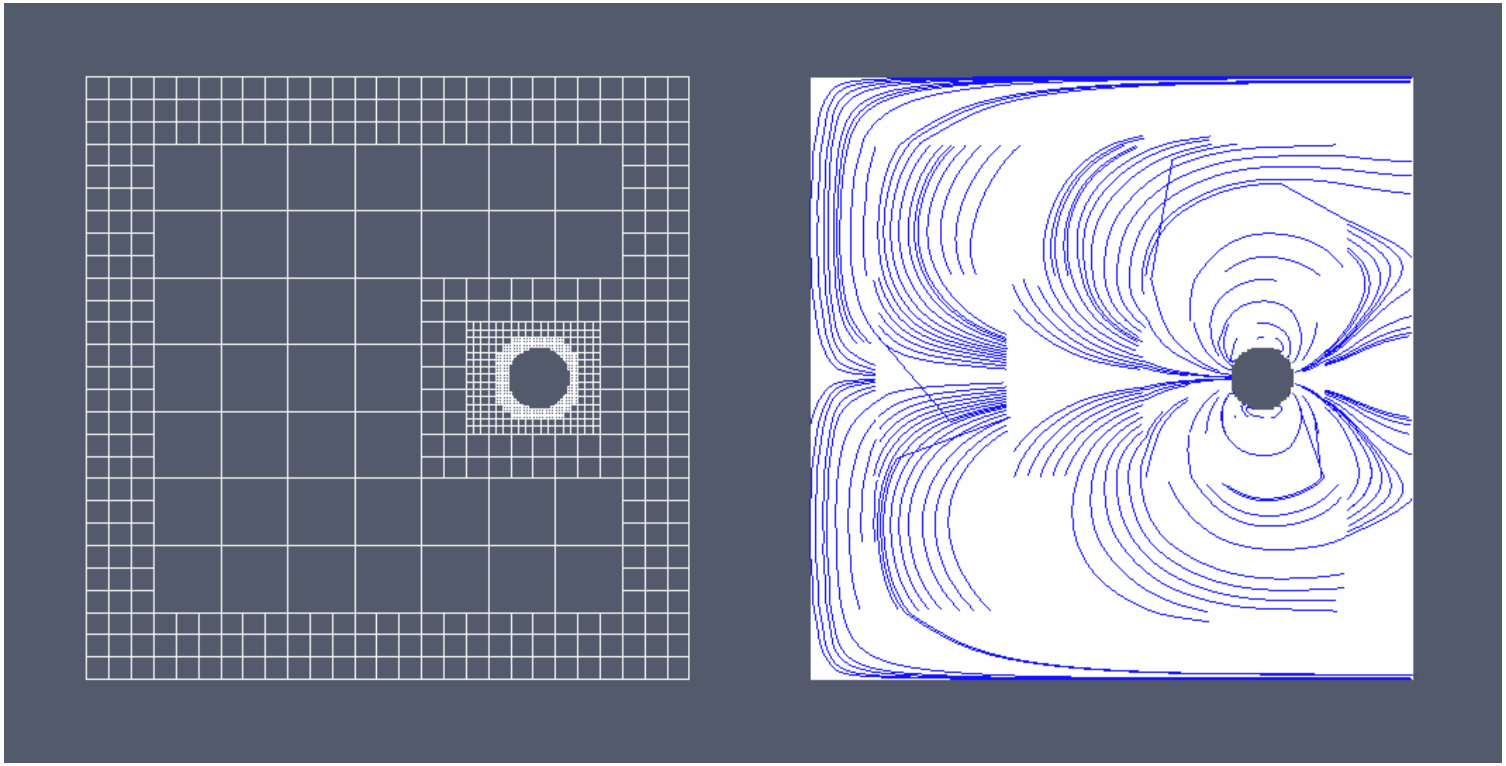
2D FEM



3D FEM — Overhead adaptive vs. regular **< 3%**

2D IDO — Overhead Peano vs. Aoki (regular): **1.3 – 4.4**

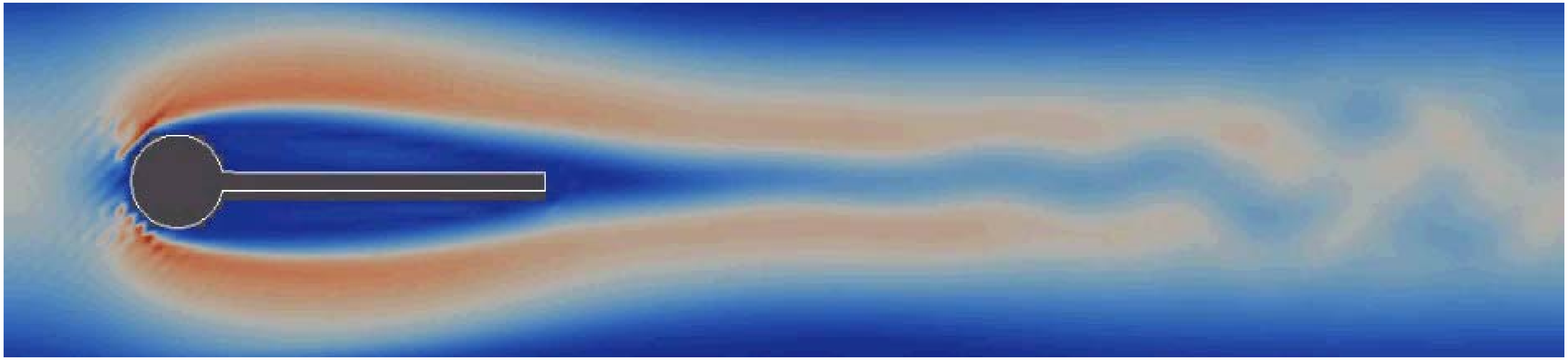
CFD Extensions



joint work with Kristof Unterweger

- Moving geometries
  - Update of data + grid (regular + adaptive)
  - Divergence correction

CFD Extensions



joint work with Janos Benk, Bernhard Gatzhammer, Miriam Mehl, Kristof Unterweger, and Tobias Weinzierl

- Moving geometries
  - Update of data + grid (regular + adaptive)
  - Divergence correction