

# Spatial Discretisation Schemes in the PDE framework Peano for Fluid-Structure Interactions

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IMECE 2010, Vancouver, November 16, 2010



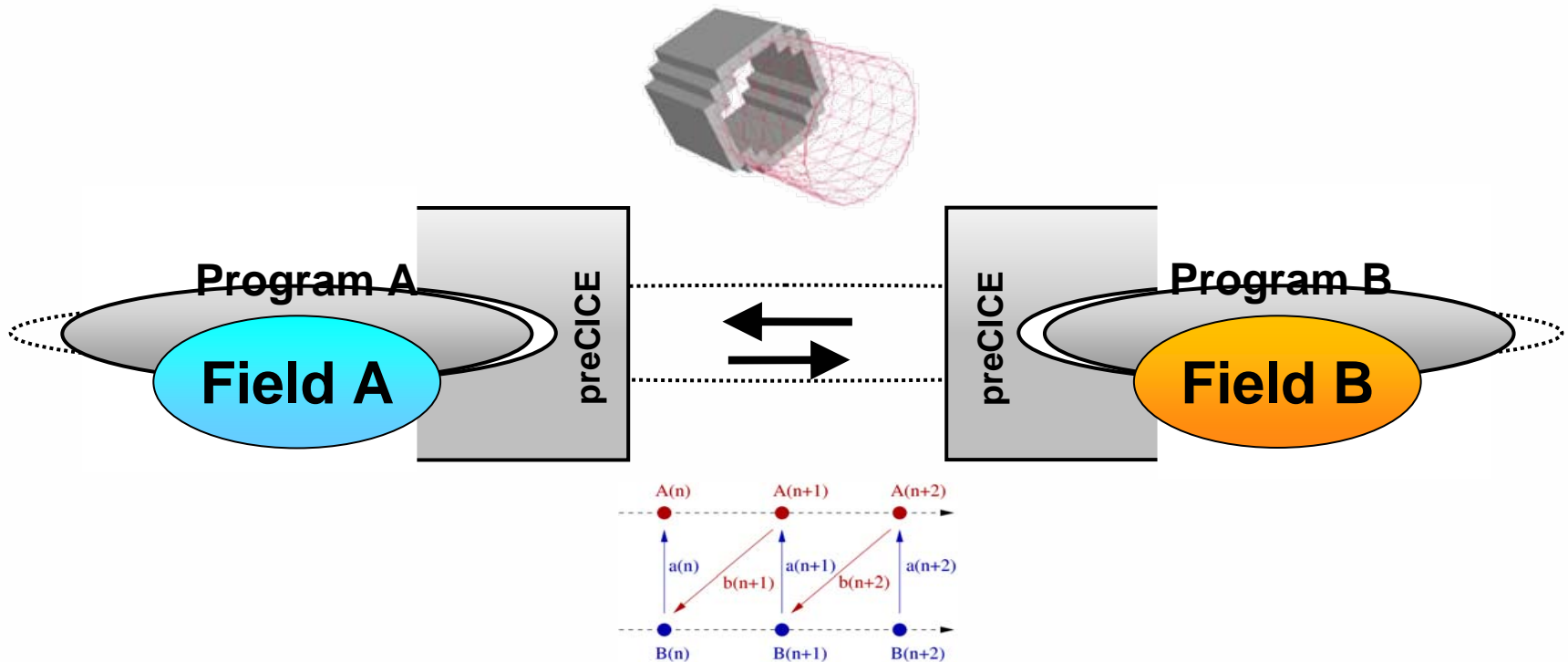
# Outline

- Introduction
  - our FSI approach
- Div-free Elements
  - Derivation
  - Driven Cavity
  - Flow around a Cylinder
- Enhanced Div-free Elements
- Outlook



# Partitioned FSI using preCICE

- no physical central control unit, local preCICE controllers instead
- P2P solver communication



## preCICE – Features

### Coupling schemes:

- Staggered explicit
- Implicit with
  - constant under-relaxation
  - Aitken- based under-relaxation
  - IQN-ILS acceleration
- Convergence measurements
- Sub-cycling
- Checkpointing

### Coupled solvers:

- CFD
  - Peano (in-house)
  - Fluent (commercial)
- CSD
  - AdhoC (academic, E. Rank et al.)
  - Structure0815 (simple in-house)
  - Comsol Multiphysics (commercial)

### Conservative/consistent linear projections data mapping



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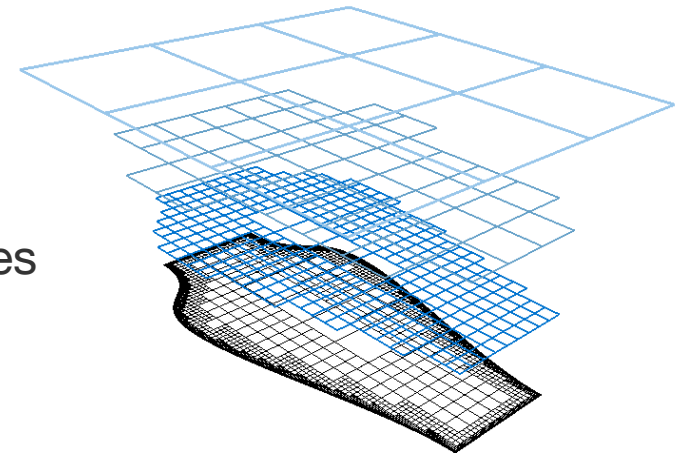
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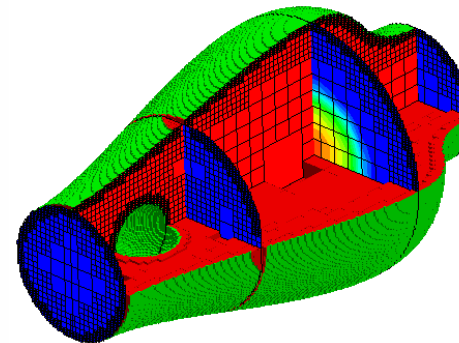


# The PDE Framework Peano

- Cartesian grids  
(recursive adaptivity, full grid hierarchy)
- Low memory requirements
- Space-Filling curves + stack data structures  
→ high cache-hit rates (>98%)
- Shared/distributed mem. parallelisation
  
- Software engineering aspects



source: T. Weinzierl



## Three simple FSI examples

- Video 1: flexible sphere in channel flow
- Video 2: 2D cantilever benchmark

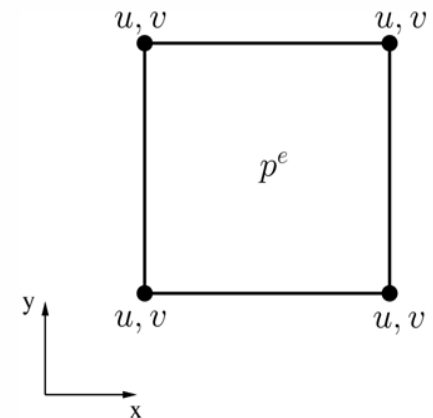


# CFD – Spatial Discretisation

- Incompressible Navier-Stokes Equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \frac{1}{\rho} \nabla p - \nu \Delta \mathbf{u} = \mathbf{0} \quad \nabla \cdot \mathbf{u} = 0$$

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  - low-order FEM (Q1Q0, etc.)





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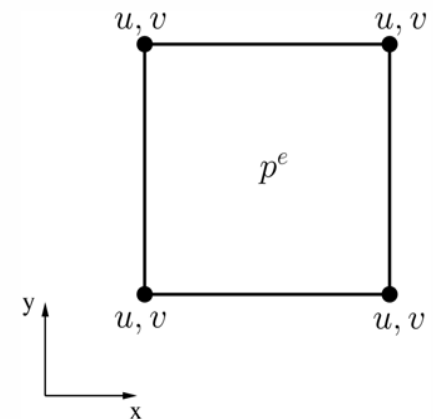
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- Idea 1: enlarge elements to avoid CB modes

- Idea 2: divergence-free elements:

- solenoidal velocity field in **every** point in a discrete cell
- simultaneous conservation of momentum **AND** energy



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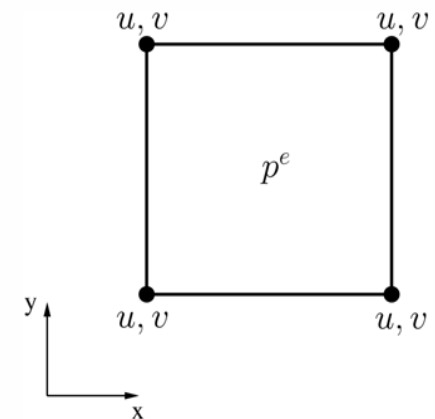
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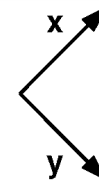
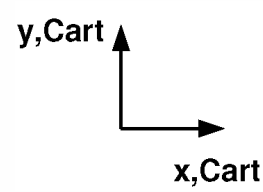
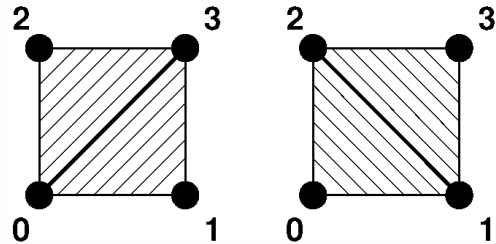
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} Combine

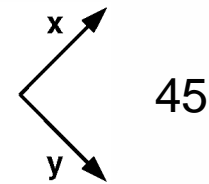
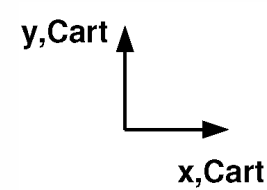
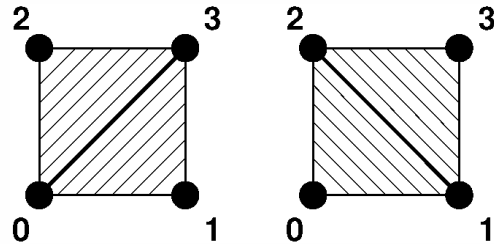
# Div-free Elements – Derivation



45



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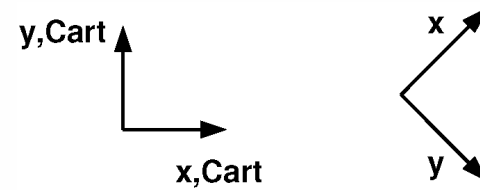
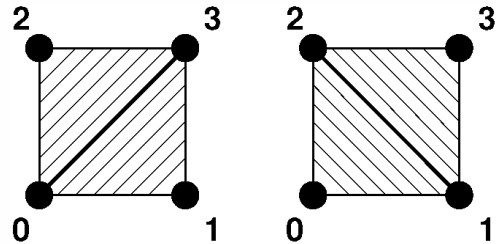


$$du/dx = \text{const} = u_3 - u_0$$

$$dv/dy = \text{const} = v_1 - v_2$$

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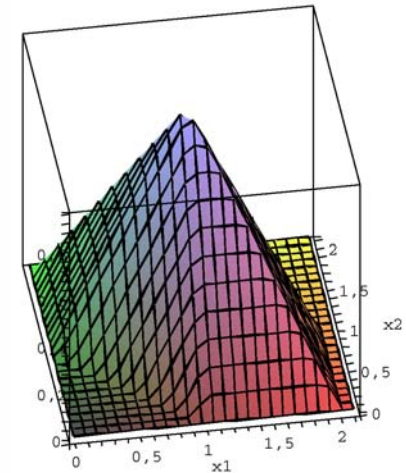
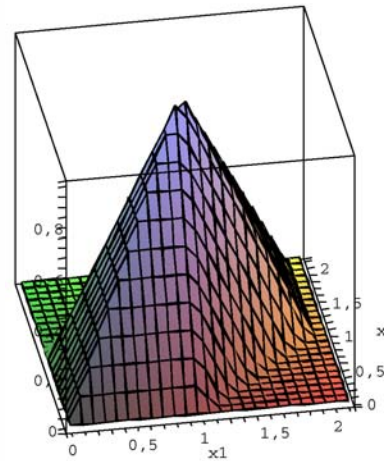


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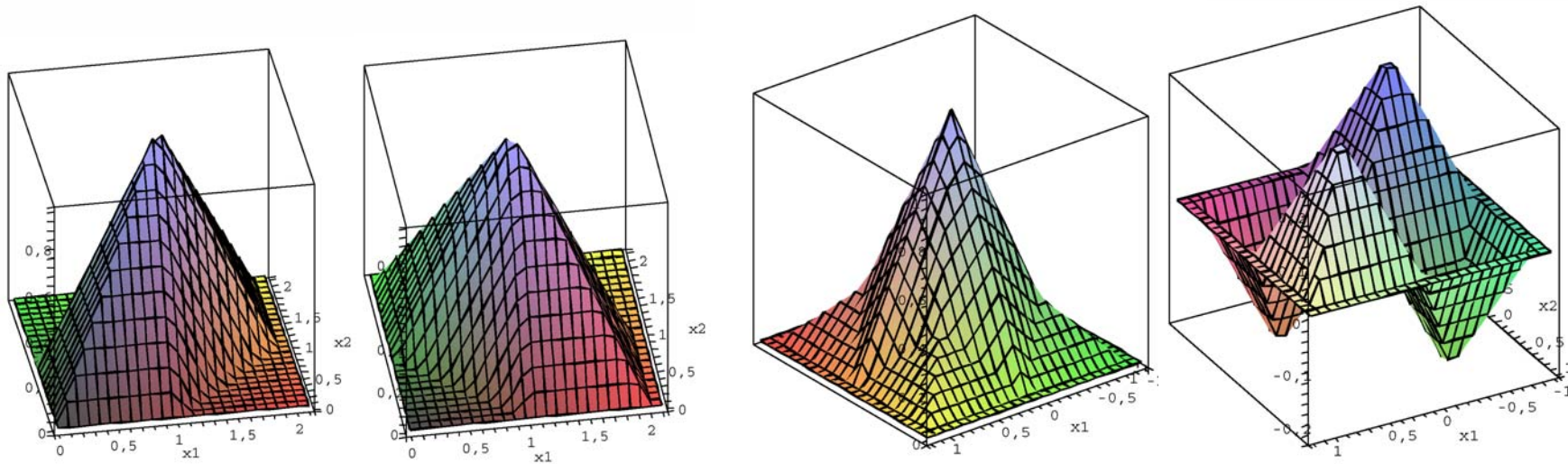
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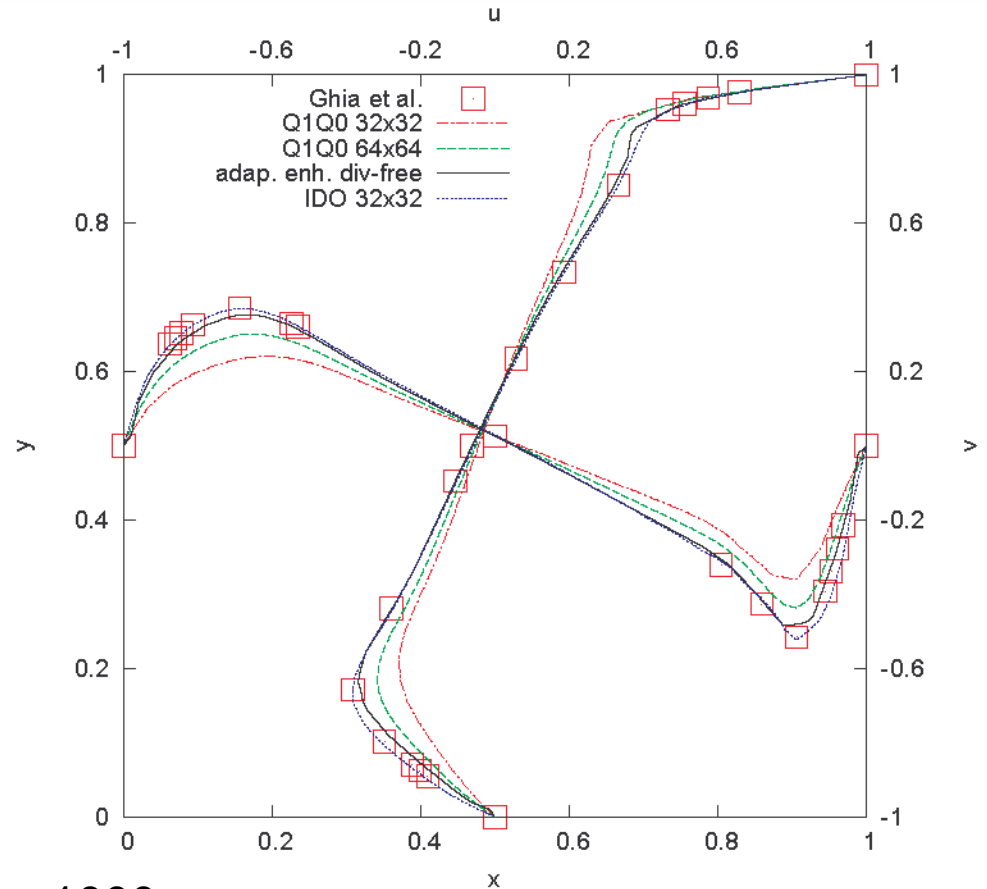
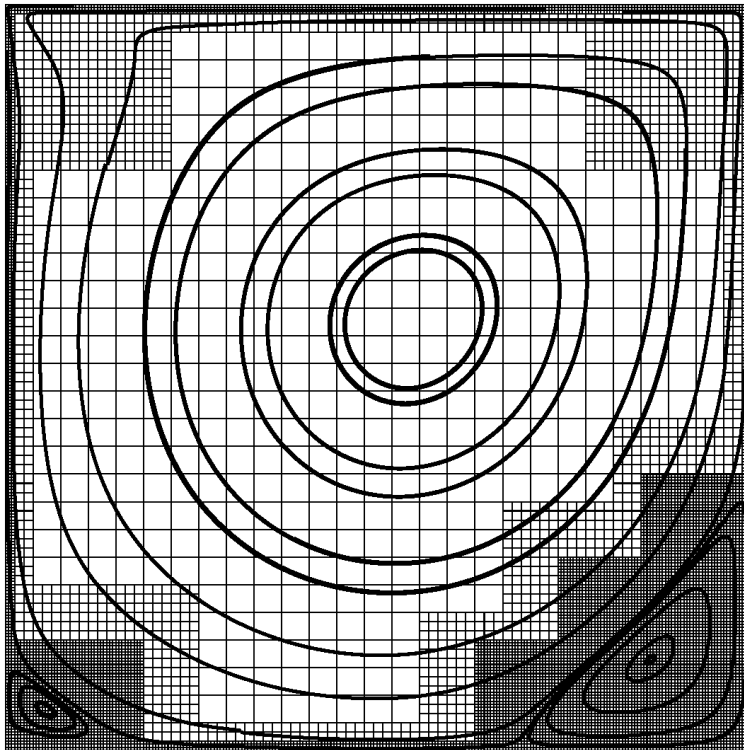
## Div-free Elements – Derivation II

- Ansatz functions 45 and Cartesian:



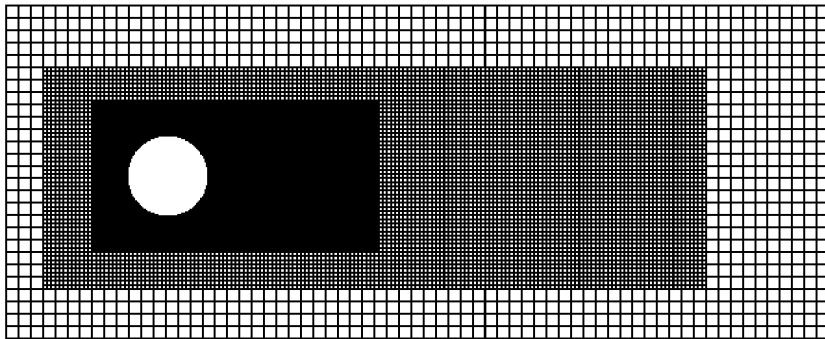
- Advantages of 45 representation:
  - Simplified derivation and representation of elements
  - Performance: ~20% less runtime for evaluation of operators  $D$  and  $C$

# Div-free Elements – Driven Cavity

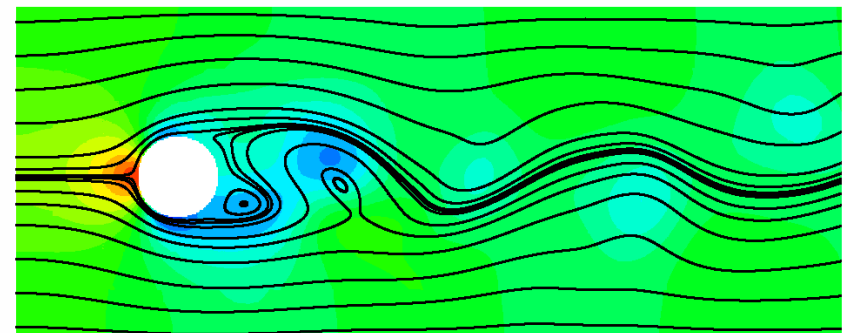
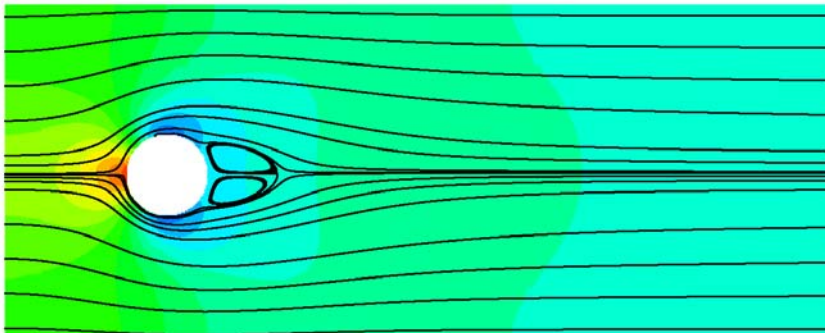


Re=1000

# Div-free Elements – Flow around a Cylinder



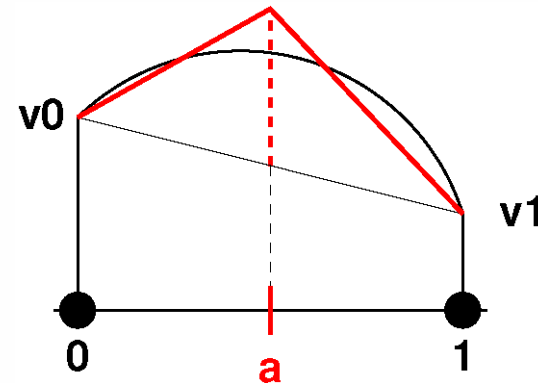
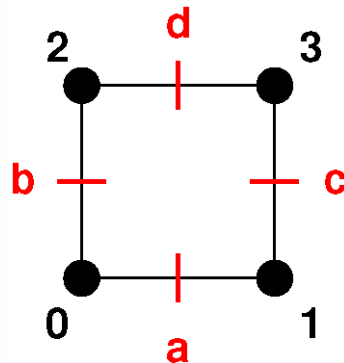
# DoF	Re = 20		Re = 100		
	$C_d$	$C_l$	$C_{d,max}$	$C_{l,max}$	St
88,857	5.68	0.0151	3.225	0.94	0.299
ref.	5.58	0.0107	3.230	1.00	0.298



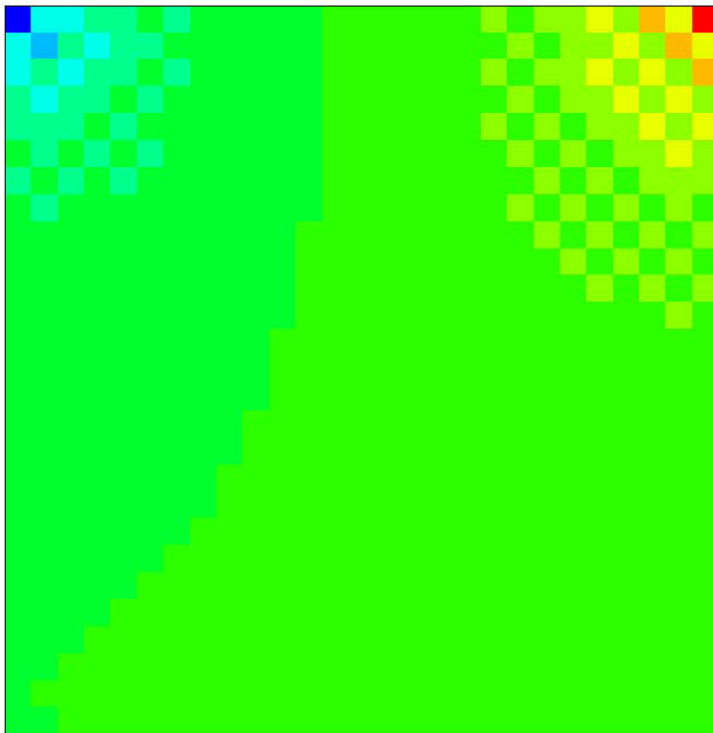


## Enhanced Div-free Elements

- Additional DoF on faces:
  - exact representation of fluxes on edges
  - no checkerboarding

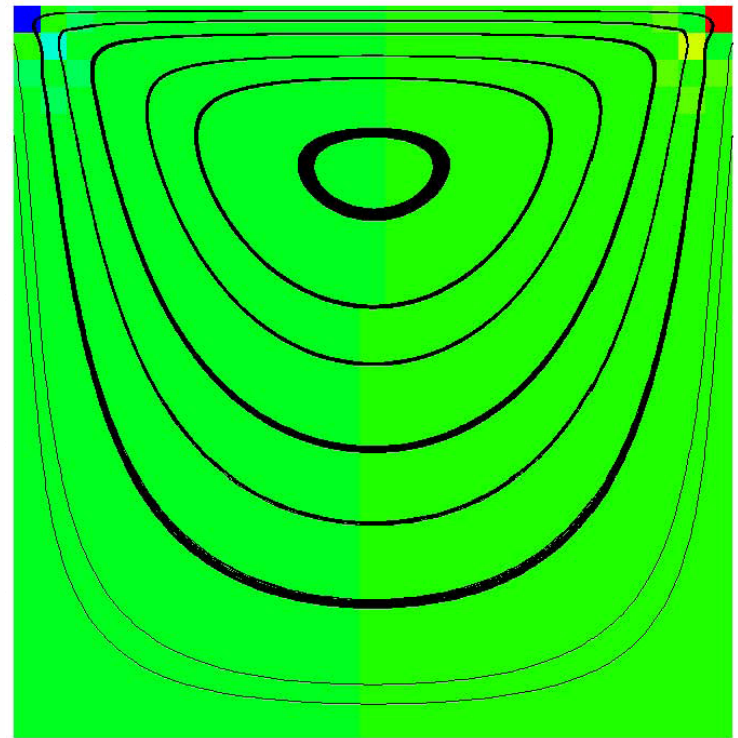


# Enh. Div-free Elements – Checkerboard Driven Cavity



Q1Q0  
(step1, **no convergence!**)

Re=1



enhanced div-free  
(steady state)

# Outlook

- Adaptive enhancement
- Multigrid
  - Peano Framework designed for hierarchical applications
  - Speed up computations while keeping low memory requirements
- Extension of (enhanced) div-free elements to 3D



**Thanks for your attention!**

