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Random Ordinary Differential Equations for Multi-Storey Buildings

Goal: Earthquake + buildings

- Earthquake excitation via time-dependent model
- Simple + realistic approach to stochastic features

➔ **Kanai-Tajimi + RODE + Wireframe buildings**

Random & Stochastic Ordinary Differential Equations:

- RODE:

$$\frac{dX_t}{dt} = f(X_t(.), t, \omega), \qquad X_t(.) \in R^d$$

Doss-Sussmann/Imkeller-Schmalfuss

- SODE:

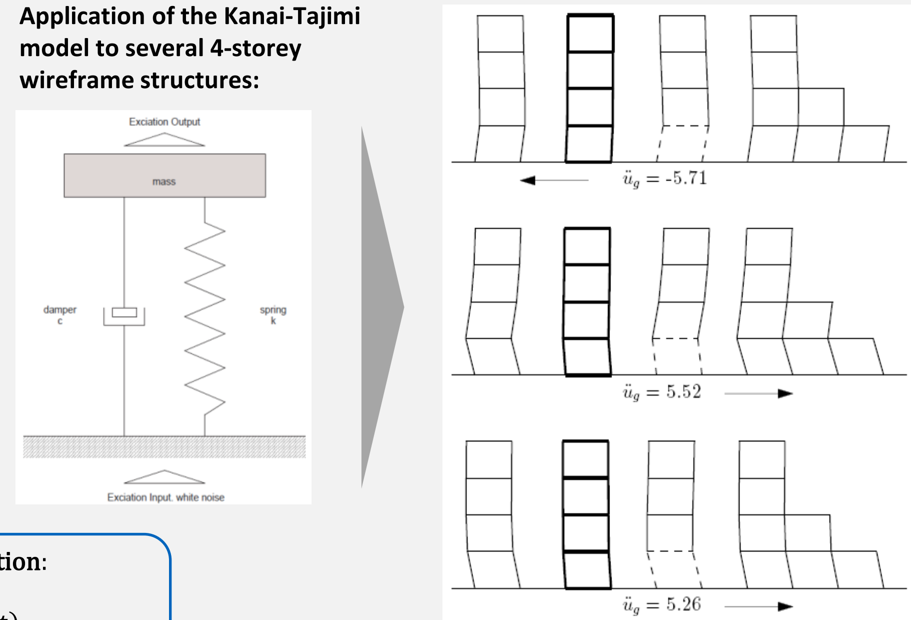
$$dX_t = a(X_t, t)dt + b(X_t, t)dW_t$$

➔ Identical trajectories but different noise!

Pathwise solution:

$$\frac{dx}{dt} = F_\omega(x,t) = G(t) + g(t)H(x)$$

Application of the Kanai-Tajimi model to several 4-storey wireframe structures:



Kanai-Tajimi Earthquake Model:

Ground motion acceleration $\ddot{u}_g = \ddot{x}_g + \xi_t$ where $\ddot{x}_g$ is the solution of

$$\ddot{x}_g + 2\zeta_g\omega_g\dot{x}_g + \omega_g^2 x_g = -\xi_t, \qquad x_g(0) = \dot{x}_g(0) = 0$$

and ξ_t is a zero-mean Gaussian white noise.

➔ Formulation as RODE (1st order):

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} -(z_2 + O_t) \\ -2\zeta_g\omega_g(z_2 + O_t) + \omega_g^2 z_1 + O_t \end{pmatrix}$$

where O_t is the Ornstein-Uhlenbeck stochastic process.

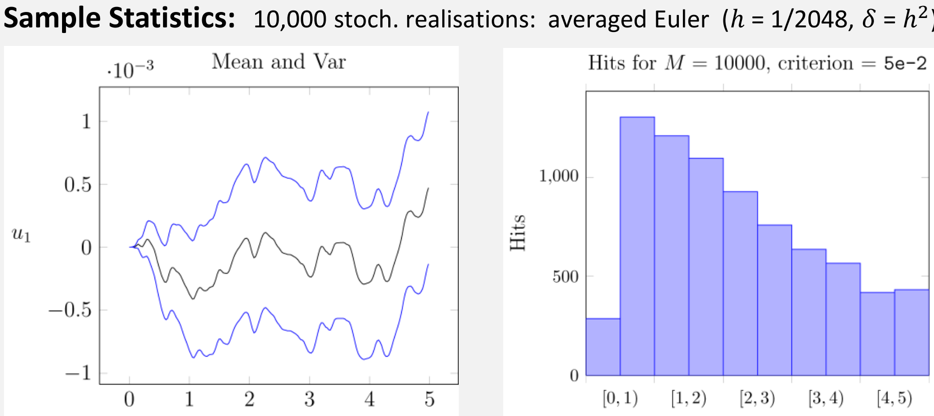
Choice of parameters: $\zeta = 0.64$, $\omega = 15.56$ rad/sec.

Wireframe Model for Buildings:

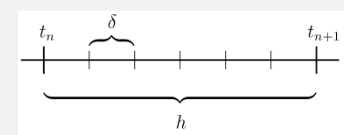
Classical spring-damper system:

$$\ddot{u} + C\dot{u} + Ku = F(t), \qquad K = \begin{pmatrix} k_1 + k_2 & -k_2 & & & \\ -k_2 & k_2 + k_3 & -k_3 & & \\ & -k_3 & k_3 + k_4 & -k_4 & \\ & & \ddots & \ddots & \\ & & & -k_n & k_n \end{pmatrix}$$

Sample Statistics: 10,000 stoch. realisations: averaged Euler ($h = 1/2048$, $\delta = h^2$)



Numerical RODE Simulations:



Problem: Order of convergence demands **smoothness** of r.h.s.!

Solution: Subcycling $\delta = h/N$ and Averaging $\bar{g}^1_{h,\delta}(t) = \frac{1}{N}\sum_{j=0}^{N-1} g(t + j\delta)$

$$\bar{g}^2_{h,\delta}(t) = \frac{2}{N^2}\sum_{i=0}^{N-1}\sum_{j=0}^{i} g(t + j\delta)$$

Wireframe Building: h

- Euler

Kanai-Tajimi: δ

- Averaged Euler

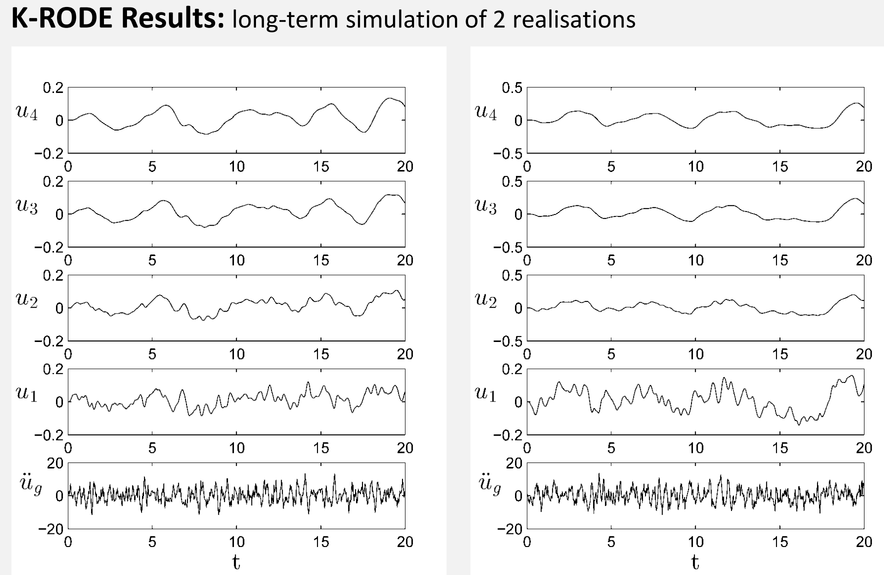
$$x_{n+1} = x_n + h\bar{G}^1_{h,\delta}(t_n) + h\bar{g}^1_{h,\delta}(t)H(x_n)$$

- Heun
- Averaged Heun

$$x_{n+1} = x_n + h\bar{G}^1_{h,\delta}(t_n) + \frac{h}{2}\bar{g}^1_{h,\delta}(t)\left[H(x_n) + H\left(x_n + h\bar{G}^2_{h,\delta}(t_n) + h\bar{g}^2_{h,\delta}(t)H(x_n)\right)\right]$$

- Runge-Kutta
- K-RODE Taylor schemes (recursively defined)

K-RODE Results: long-term simulation of 2 realisations



Literature:

- L. Grüne and P.E. Kloeden: *Pathwise Approximation of Random Ordinary Differential Equations*, BIT Numerical Mathematics, 41(4), pp. 711-721, 2001
- D.T. Gillespie: *Exact numerical simulation of the Ornstein-Uhlenbeck process and its integral*, Physical Review E, 54(2), pp. 2084-2091, 1996
- A. Jentzen and P.E. Kloeden: *Taylor Approximations for Stochastic Partial Differential Equations*, CBMS-NSF Conf. Series in Applied Math. (No.83), SIAM Press, 2011

Extension:

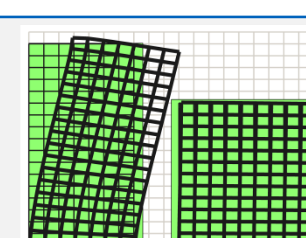
- RPDE:

$\rho\, u_{tt}(x, t) = \text{div}(\ \sigma(\ u(x, t)\ )\ ) + \beta(x, t)$

$u(x, t) = u_0(x, t)$

$\sigma(\ u(x, t)\ )\ n(x) = \tau(x, t)$

- Hardware platforms (vectorisation)



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