Random Ordinary Differential Equations for Multi-Storey Buildings

Goal: Earthquake + buildings
- Earthquake excitation via time-dependent model
- Simple + realistic approach to stochastic features

→ Kanai-Tajimi + RODE + Wireframe buildings

Random & Stochastic Ordinary Differential Equations:
- RODE:
  \[
  \frac{dX_t}{dt} = f(X_t(\cdot), t, \omega), \quad X_t(\cdot) \in \mathbb{R}^d
  \]
  Doss-Sussmann/Imkeller-Schmalfuss
- SODE:
  \[
  dX_t = a(X_t, t)dt + b(X_t, t)dW_t
  \]

→ Identical trajectories but different noise!

Kanai-Tajimi Earthquake Model:
Ground motion acceleration \( \ddot{u}_g = \ddot{x}_g + \dot{\xi}_t \) where \( \ddot{x}_g \) is the solution of
\[
\ddot{x}_g + 2\xi_g \omega_g \dot{x}_g + \omega_g^2 x_g = -\dot{\xi}_t, \quad x_g(0) = \ddot{x}_g(0) = 0
\]
and \( \dot{\xi}_t \) is a zero-mean Gaussian white noise.

→ Formulation as RODE (1st order):
\[
\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} -2\xi_g \omega_g (z_2 + O_t) + \omega_g^2 z_1 + O_t \\ -\omega_g \dot{\xi}_t \end{pmatrix}
\]
where \( O_t \) is the Ornstein-Uhlenbeck stochastic process.

Choice of parameters: \( \xi = 0.64, \omega = 15.56 \text{ rad/sec.} \)

Wireframe Model for Buildings:
Classical spring-damper system:
\[
\ddot{u} + C\dot{u} + Ku = f(t), \quad K = \begin{pmatrix} k_1 & -k_2 & \cdots & -k_n \\ -k_2 & k_3 & \cdots & -k_n \\ \vdots & \vdots & \ddots & \vdots \\ -k_n & k_{n-1} & \cdots & k_1 \end{pmatrix}
\]

Sample Statistics: 10,000 stoch. realisations: averaged Euler \( \delta h = 1/2048 \)

Application of the Kanai-Tajimi model to several 4-storey wireframe structures:

Pathwise solution:
\[
\frac{dx}{dt} = F_u(x, t) + G(t)H(x)
\]

Numerical RODE Simulations:
- Problem: Order of convergence demands smoothness of r.h.s.!
- Solution: Subcycling \( \delta = h/N \) and Averaging
  \[
  \bar{g}_{h,\delta}(t) = \frac{1}{N} \sum_{j=0}^{N-1} g(t + j\delta)
  \]
- Wireframe Building: \( h \)
  - Euler
  - Averaged Euler
  - Heun
  - Averaged Heun
  - Runge-Kutta
  - K-RODE Taylor schemes (recursively defined)

K-RODE Results: long-term simulation of 2 realisations

Literature:

Extension:
- RPDE:
  \[
  \rho \dot{u}_0(x, t) = \text{div} \left( \sigma(u(x, t)) \right) + \beta(x, t)
  \]
  \( \sigma(u(x, t)) = u(x, t) \)
  \( \beta(x, t) \)
- Hardware platforms (vectorisation)

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