SIMULATION OF THERMOHYDRAULIC PHENOMENA IN THE PDE FRAMEWORK PEANO

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Theory

Navier-Stokes equations using Boussinesq approximation

- continuity equation
  \[ \nabla \cdot \mathbf{u} = 0 \]
- momentum equation
  \[ \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \mu \nabla^2 \mathbf{u} \]
- energy equation
  \[ \frac{\partial \mathbf{T}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{T} = \alpha \nabla^2 \mathbf{T} \]

Peano’s degrees of freedom

- velocity \( \mathbf{u} : \Omega \times [t_i, t_f] \rightarrow \mathbb{R}^d \)
- pressure \( p : \Omega \times [t_i, t_f] \rightarrow \mathbb{R} \)
- temperature \( T : \Omega \times [t_i, t_f] \rightarrow \mathbb{R} \)
- domain \( \Omega \subset \mathbb{R}^d \)
- time \( t \in [t_i, t_f] \)

Boussinesq approximation

- density constant except in buoyancy terms
- linear relation between density and temperature
- all other fluid parameters constant
- viscous dissipation negligibly small

Implementation

PDE Framework Peano

- memory efficient, stack-based \( O^+ \) framework developed at TUM
- regular and adaptively refined Cartesian grids using space-filling Peano curve
- cell-wise operator evaluation
- CFD component (without energy) using low order FEM (linear velocity, piecewise constant pressure)
- explicit and implicit time stepping

Peano’s chemical component

- extension of Peano’s CFD component
- additional vertex attributes (\( \mathbf{T}, \mathbf{HeatScenario}, \mathbf{HeatScenarioVertexType} \))
- same discretization as for CFD component
- only one additional grid traversal for energy equation (results in one class)
- explicit Euler method for temperature update

Validation

Natural Convection with Heated Lateral Walls

- box with heated \( (T_H) \) and cooled \( (T_C) \) lateral walls (Fig. 1)
- adiabatic walls everywhere else
- gravity involved (buoyancy forces: heated fluid rises)
- current depending on fluid parameters
- diffusive (Fig. 2), convective heat transport (Fig. 3a, 3b and 4)

Rayleigh-Bénard Convection

- box with heated \( (T_H) \) and cooled \( (T_C) \) walls (Fig. 5)
- adiabatic walls everywhere else
- occurrence and analysis of Rayleigh-Bénard cells (Fig. 6)

Flat Plate in Parallel Flow

- flat heated plate in global flow (Fig. 7)
- external inflow at constant speed \( (u_\infty) \)
- slip-wall velocity boundary condition at top of simulation domain
- velocity and thermal boundary layer analysis for water

Application to Reactor Safety and Outlook

Simulation of Flow in Cold Leg

- recirculating power plant, light water reactor (Fig. 8)
- water as coolant and moderator
- primary cooling circuit (approx. 310 °C)
- single-phase flow due to high pressure (approx. 155 bar)
- cold leg with nozzle for emergency core coolant (ECC) (approx. 30 °C)
- injection of ECC in loss-of-coolant accident
- first simulations (Fig. 9) using provided technical base (Fig. 10)

Outlook

- thorough analysis of thermal boundary layers in cold leg using UPTF-TRAM test series for validation
- simulation of complete primary cooling circuit (in emergency)
- fully reached 3D phenomena in specific components of the cooling circuit (e.g., turbulence)
- fully automatic dynamic mesh refinement
- parallelization (distributed and shared memory)
- long-term goals:
  - coupling 3D finite volume of complete cooling circuit
  - two-phase simulations