**Algorithms of Scientific Computing**

*Change of Schedule:* due to the student assembly on Wednesday, the tutorial will replace the lecture on Thursday, 10-12, in room MI 01.10.011.

**Exercise 1**

The functions cos and sin are axially respectively point symmetric to the ascension of 180 degrees. What can be found for the coefficients $a_k$ and $b_k$ from the last worksheet, if the following conditions hold:

$$X_l = X(\theta_l) = X(360 - \theta_l) = X_{12-l}$$ respectively
$$X_l = X(\theta_l) = -X(360 - \theta_l) = -X_{12-l}$$

Hint: Which values are allowed for $X_0$ and $X_6$ in the case $X_l = -X_{12-l}$?

**Exercise 2**

In the last worksheet we showed that the $a_k$ and $b_k$ can be computed by

$$c_k = \frac{1}{12} \sum_{l=0}^{11} X_l e^{-i2\pi kl/12},$$

i.e. by a DFT.

Use the idea of the Fast Fourier Transformation, to reduce this DFT of length 12 to the computation of some DFTs of length 6 or 3, respectively.

Use the fact that all $X_l \in \mathbb{R}$.

Draw a diagram, that shows the needed computation steps or write an appropriate program (for example in Python).
Exercise 3: DFT of Mirrored data

Assume a dataset $f_n$, $n = 0, \ldots, N - 1$. What is the difference of the Fourier coefficients for this dataset and the “mirrored” dataset $\tilde{f}_n := f_{N-n}$?

Exercise 4: DFT and “Padding”

A dataset $f_n$, $n = 0, \ldots, N - 1$ is extended by “zeros”, which gives the dataset $\hat{f}_n$, $n = 0, \ldots, M - 1$, with

$$
\hat{f}_n := \begin{cases} 
  f_n & \text{if } n \leq N - 1 \\
  0 & \text{if } N \leq n \leq M - 1
\end{cases}
$$

What is the difference between the Fourier coefficients of the original dataset $f_n$ and the Fourier coefficients of the extended one $\hat{f}_n$?